

GCSE Maths: Reasoning and proof with fractions (Higher): answers

GCSE Maths - Fractions - Higher - Lesson 3.6

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at websink.site/gcse/maths/ch03-fractions/higher/reasoning-and-proof-with-fractions/

Exercise 3F Reasoning and proof Non-calculator

1 Show that $\frac{1}{n} + \frac{1}{n+1} = \frac{2n+1}{n(n+1)}$.

Answer: $\frac{n+1}{n(n+1)} + \frac{n}{n(n+1)} = \frac{2n+1}{n(n+1)}$

2 Work out $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \dots + \frac{1}{19 \times 20}$.

Answer: $1 - \frac{1}{20} = \frac{19}{20}$

3 Prove that $0.\dot{1}8 = \frac{2}{11}$.

Answer: Let $x = 0.1818\dots$. Then $100x = 18.1818\dots$. Subtract: $99x = 18$, $x = \frac{18}{99} = \frac{2}{11}$.

4 Show that $\frac{3}{5}$ lies between $\frac{1}{2}$ and $\frac{2}{3}$. Explain how adding the numerators and adding the denominators of $\frac{1}{2}$ and $\frac{2}{3}$ gives it.

Answer: $\frac{3}{5} = 0.6$; $\frac{1}{2} = 0.5$ and $\frac{2}{3} = 0.667$ so $0.5 < 0.6 < 0.667$. $(1+2)/(2+3) = \frac{3}{5}$. This always gives a fraction between two fractions.

5 Show that $\frac{n+1}{n} - \frac{n}{n+1} = \frac{2n+1}{n(n+1)}$.

Answer: $\frac{(n+1)^2 - n^2}{n(n+1)} = \frac{2n+1}{n(n+1)}$

6 Ben says "The sum of two proper fractions is always a proper fraction." Give a counter-example.

Answer: $\frac{3}{4} + \frac{2}{3} = 1\frac{5}{12}$, which is greater than 1.

7 Write 1 as the sum of three different unit fractions.

Answer: $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3}{6} + \frac{2}{6} + \frac{1}{6} = 1$

8 **Challenge.** Find the positive whole numbers a and b with $a < b$ such that $\frac{1}{a} + \frac{1}{b} = \frac{1}{2}$.

Answer: $a = 3$, $b = 6$ (the only solution with $a < b$). Check: $1/3 + 1/6 = 1/2$.