

GCSE Maths: Roots and fractional indices (Higher): answers

GCSE Maths - Standard Form, Indices and Surds - Higher - Lesson 6.3

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at websink.site/gcse/maths/ch06-standard-form-indices-surds/higher/roots-and-fractional-indices/

Exercise 6E Roots without a calculator Non-calculator

1 Work out each square root.

(a) $\sqrt{36}$

Answer: 6

(b) $\sqrt{81}$

Answer: 9

(c) $\sqrt{121}$

Answer: 11

(d) $\sqrt{169}$

Answer: 13

(e) $\sqrt{400}$

Answer: 20

(f) $\sqrt{25}$

Answer: 5

2 Work out each cube root.

(a) $\sqrt[3]{8}$

Answer: 2

(b) $\sqrt[3]{27}$

Answer: 3

(c) $\sqrt[3]{64}$

Answer: 4

(d) $\sqrt[3]{125}$

Answer: 5

(e) $\sqrt[3]{1000}$

Answer: 10

(f) $\sqrt[3]{216}$

Answer: 6

3 Work out each square root.

(a) $\sqrt{\frac{9}{16}}$

Answer: $\frac{3}{4}$

(b) $\sqrt{\frac{4}{25}}$

Answer: $\frac{2}{5}$

(c) $\sqrt{\frac{1}{36}}$

Answer: $\frac{1}{6}$

(d) $\sqrt{\frac{49}{64}}$

Answer: $\frac{7}{8}$

4 Work out each root. Give your answer as a decimal.

(a) $\sqrt{0.25}$

Answer: 0.5

(b) $\sqrt{0.81}$

Answer: 0.9

(c) $\sqrt{0.0144}$

Answer: 0.12

(d) $\sqrt[3]{0.027}$

Answer: 0.3

(e) $\sqrt[3]{0.008}$

Answer: 0.2

5 Work out each calculation.

(a) $\sqrt{49} + \sqrt{81}$

Answer: 16

(b) $\sqrt[3]{64} - \sqrt{16}$

Answer: 0

(c) $5^2 - \sqrt[3]{125}$

Answer: 20

(d) $\sqrt{100} + 3^3 - \sqrt{4}$

Answer: 35

6 Between which two whole numbers does each root lie?

(a) $\sqrt{20}$

Answer: 4 and 5

(b) $\sqrt{70}$

Answer: 8 and 9

(c) $\sqrt[3]{30}$

Answer: 3 and 4

(d) $\sqrt[3]{100}$

Answer: 4 and 5

(e) $\sqrt{200}$

Answer: 14 and 15

7 Solve each equation.

(a) $x^2 = 49$

Answer: $x = \pm 7$

(b) $x^2 = 121$

Answer: $x = \pm 11$

(c) $x^3 = 27$

Answer: $x = 3$

(d) $x^3 = 125$

Answer: $x = 5$

8 Work out each value.

(a) $(\sqrt{7})^2$

Answer: 7

(b) $(\sqrt{13})^2$

Answer: 13

(c) $(\sqrt{2})^2$

Answer: 2

(d) $(\sqrt{50})^2$

Answer: 50

9 A square patio has an area of 81 m^2 . Work out the length of one side, and the length of the fence needed to go all the way around the patio.

Answer: Side 9 m; fence 36 m

10 A cube-shaped box has a volume of 216 cm^3 . Work out the total length of all 12 edges of the box.

Answer: 72 cm

- 11** A square has an area of 150 cm^2 . Between which two whole numbers is the length of its side?

Answer: 12 and 13 cm

- 12** Is $\sqrt{9 + 16}$ the same as $\sqrt{9} + \sqrt{16}$? Show your working.

Answer: No: $\sqrt{25} = 5$ but $3 + 4 = 7$

- 13** Show that $\sqrt{50}$ is greater than 7 but less than 7.1.

Answer: $7^2 = 49 < 50 < 50.41 = 7.1^2$

- 14** Work out $\sqrt[3]{-8}$ and $\sqrt[3]{-27}$.

(a) $\sqrt[3]{-8}$

Answer: -2

(b) $\sqrt[3]{-27}$

Answer: -3

- 15** Explain why $\sqrt{-9}$ cannot be worked out using real numbers.

Answer: A positive number squared is positive and a negative number squared is also positive, so no real number squares to give -9.

- 16** Find a number between 50 and 100 that is both a square number and a cube number.

Answer: 64

Exercise 6F Powers and roots with a calculator Calculator allowed

- 1** Use a calculator to work out each root. Give your answers to 3 significant figures.

(a) $\sqrt{7}$

Answer: 2.65

(b) $\sqrt{130}$

Answer: 11.4

(c) $\sqrt{2000}$

Answer: 44.7

(d) $\sqrt[3]{50}$

Answer: 3.68

(e) $\sqrt[3]{300}$

Answer: 6.69

- 2** Use a calculator to work out each power. Give your answers to 3 significant figures.

(a) 2.5^4

Answer: 39.1

(b) 1.05^{10}

Answer: 1.63

(c) 3.2^{-2}

Answer: 0.0977

(d) 0.9^5

Answer: 0.590

(e) 8^{-2}

Answer: 0.0156

- 3 A square picture has an area of 90 cm^2 . Work out the length of one side to 1 decimal place.

Answer: 9.5 cm

- 4 £1500 is invested. After 5 years it is worth $£1500 \times 1.04^5$. Work out its value, to the nearest penny.

Answer: £1,824.98

- 5 Work out $\sqrt{5.2^2 + 3.9^2}$.

Answer: 6.5

- 6 Use a calculator to work out each value. Give your answers to 3 significant figures.

(a) $\sqrt{11}$

Answer: 3.32

(b) $\sqrt{59}$

Answer: 7.68

(c) 1.5^6

Answer: 11.4

(d) 0.5^{-3}

Answer: 8.00

- 7 Show, using a calculator, that $\sqrt[3]{100}$ is between 4.6 and 4.7.

Answer: $4.6^3 = 97.336 < 100 < 103.823 = 4.7^3$

Exercise 6G Fractional indices

Non-calculator

- 1 Work out each value.

(a) $25^{1/2}$

Answer: 5

(b) $81^{1/2}$

Answer: 9

(c) $64^{1/3}$

Answer: 4

(d) $125^{1/3}$

Answer: 5

(e) $16^{1/4}$

Answer: 2

(f) $32^{1/5}$

Answer: 2

2 Work out each value.

(a) $8^{2/3}$

Answer: 4

(b) $9^{3/2}$

Answer: 27

(c) $16^{3/4}$

Answer: 8

(d) $27^{2/3}$

Answer: 9

(e) $4^{5/2}$

Answer: 32

(f) $125^{2/3}$

Answer: 25

3 Work out each value.

(a) $4^{-1/2}$

Answer: $\frac{1}{2}$

(b) $8^{-2/3}$

Answer: $\frac{1}{4}$

(c) $27^{-1/3}$

Answer: $\frac{1}{3}$

(d) $16^{-3/4}$

Answer: $\frac{1}{8}$

(e) $25^{-3/2}$

Answer: $\frac{1}{125}$

(f) $32^{-2/5}$

Answer: $\frac{1}{4}$

4 Work out each value.

(a) $(\frac{4}{9})^{1/2}$

Answer: $\frac{2}{3}$

(b) $(\frac{8}{27})^{2/3}$

Answer: $\frac{4}{9}$

(c) $(\frac{27}{8})^{-2/3}$

Answer: $\frac{4}{9}$

(d) $(\frac{16}{81})^{-3/4}$

Answer: $\frac{27}{8}$

5 Work out each calculation.

(a) $8^{1/3} \times 4^{1/2}$

Answer: 4

(b) $27^{2/3} \div 9^{1/2}$

Answer: 3

(c) $16^{1/4} + 2^{-1}$

Answer: $\frac{5}{2}$

6 Write each expression as a root, and each root as a power.

(a) $x^{1/2}$ as a root

Answer: $\sqrt{x}$

(b) $\sqrt[3]{y}$ as a power

Answer: $y^{1/3}$

(c) $5^{2/3}$ as roots and powers

Answer: $(\sqrt[3]{5})^2$

7 Eve says $9^{1/2} = 4.5$. Explain her mistake and give the correct value.

Answer: Eve halved 9. The index $1/2$ means square root: $9^{1/2} = 3$.

8 Show that $8^{2/3} \times 4^{-1/2} = 2$.

Answer: $4 \times 1/2 = 2$

9 In suitable units, the time T that a planet takes to orbit a star is $T = r^{3/2}$, where r is its distance from the star. Work out T when $r = 4, 9$ and 25 .

(a) $r = 4$

Answer: 8

(b) $r = 9$

Answer: 27

(c) $r = 25$

Answer: 125

10 Explain, using the laws of indices, why $49^{1/2}$ must equal $\sqrt{49}$.

Answer: $49^{1/2} \times 49^{1/2} = 49^1 = 49$, so $49^{1/2}$ is a number that multiplies by itself to give 49: the square root, 7.

Exercise 6H Fractional indices in algebra and equations

Non-calculator

1 Simplify each expression.

(a) $x^{1/2} \times x^{3/2}$

Answer: x^2

(b) $x^{2/3} \times x^{1/3}$

Answer: x

(c) $y^{5/2} \div y^{1/2}$

Answer: y^2

(d) $(a^{1/2})^4$

Answer: a^2

2 Simplify each expression.

(a) $(16x^8)^{3/4}$

Answer: $8x^6$

(b) $(27y^6)^{1/3}$

Answer: $3y^2$

(c) $(4a^2)^{1/2}$

Answer: $2a$

(d) $(8x^3)^{2/3}$

Answer: $4x^2$

(e) $(81a^4b^8)^{1/4}$

Answer: $3ab^2$

3 Simplify, giving your answers with positive indices.

(a) $(9x^4)^{-1/2}$

Answer: $\frac{1}{3x^2}$

(b) $(8y^6)^{-1/3}$

Answer: $\frac{1}{2y^2}$

(c) $x^{1/2} \div x^{3/2}$

Answer: $\frac{1}{x}$

4 Simplify each expression.

(a) $2x^{1/2} \times 3x^{3/2}$

Answer: $6x^2$

(b) $(25a^4)^{1/2} \times 2a^{-1}$

Answer: $10a$

(c) $(16x^2y^4)^{3/2} \div 8xy^3$

Answer: $8x^2y^3$

5 Solve each equation.

(a) $x^{3/2} = 27$

Answer: $x = 9$

(b) $x^{2/3} = 4$

Answer: $x = 8$

(c) $x^{3/4} = 8$

Answer: $x = 16$

(d) $x^{1/2} = 7$

Answer: $x = 49$

(e) $x^{1/3} = 4$

Answer: $x = 64$

6 Solve each equation.

(a) $4^x = 8$

Answer: $x = \frac{3}{2}$

(b) $9^x = 27$

Answer: $x = \frac{3}{2}$

(c) $8^x = 4$

Answer: $x = \frac{2}{3}$

(d) $25^x = 5$

Answer: $x = \frac{1}{2}$

(e) $9^x = \frac{1}{27}$

Answer: $x = -\frac{3}{2}$

(f) $4^x = \frac{1}{8}$

Answer: $x = -\frac{3}{2}$

7 Show that $(x^{1/2} + x^{-1/2})^2 = x + 2 + \frac{1}{x}$.

Answer: $x + 2 + 1/x$

8 $p = x^{1/3}$. Write x^2 in terms of p .

Answer: p^6

9 Solve $2^{2x} = 8^{x-1}$.

Answer: $x = 3$

10 Find x if $4^x \times 2^{x+1} = 1024$.

Answer: $x = 3$