

GCSE Maths: Factorising $x^2 + bx + c$ and the difference of two squares (Higher): answers

GCSE Maths - Algebra Basics - Higher - Lesson 7.7

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at websink.site/gcse/maths/ch07-algebra-basics/higher/factorising-x-bx-c-and-the-difference-of-two-squares/

Exercise 7G Factorising $x^2 + bx + c$ Non-calculator

1 Factorise each expression.

(a) $x^2 + 5x + 6$

Answer: $(x + 2)(x + 3)$

(b) $x^2 + 7x + 10$

Answer: $(x + 2)(x + 5)$

(c) $x^2 + 8x + 15$

Answer: $(x + 3)(x + 5)$

(d) $x^2 + 9x + 20$

Answer: $(x + 4)(x + 5)$

(e) $x^2 + 10x + 24$

Answer: $(x + 4)(x + 6)$

(f) $x^2 + 6x + 8$

Answer: $(x + 2)(x + 4)$

2 Factorise each expression.

(a) $x^2 - 7x + 12$

Answer: $(x - 4)(x - 3)$

(b) $x^2 - 6x + 8$

Answer: $(x - 4)(x - 2)$

(c) $x^2 - 10x + 21$

Answer: $(x - 7)(x - 3)$

(d) $x^2 - 8x + 15$

Answer: $(x - 5)(x - 3)$

(e) $x^2 - 9x + 14$

Answer: $(x - 7)(x - 2)$

3 Factorise each expression.

(a) $x^2 + 2x - 8$

Answer: $(x - 2)(x + 4)$

(b) $x^2 - x - 12$

Answer: $(x - 4)(x + 3)$

(c) $x^2 + 3x - 18$

Answer: $(x - 3)(x + 6)$

(d) $x^2 - 4x - 5$

Answer: $(x - 5)(x + 1)$

(e) $x^2 + 5x - 14$

Answer: $(x - 2)(x + 7)$

(f) $x^2 - 2x - 24$

Answer: $(x - 6)(x + 4)$

4 Fill in the missing numbers.

(a) $x^2 + 9x + 14 = (x + ?)(x + ?)$

Answer: $(x + 2)(x + 7)$

(b) $x^2 - 8x + 15 = (x - ?)(x - ?)$

Answer: $(x - 5)(x - 3)$

(c) $x^2 + x - 12 = (x + ?)(x - ?)$

Answer: $(x - 3)(x + 4)$

5 Factorise each expression.

(a) $y^2 + 8y + 12$

Answer: $(y + 2)(y + 6)$

(b) $a^2 - a - 20$

Answer: $(a - 5)(a + 4)$

(c) $p^2 - 10p + 16$

Answer: $(p - 8)(p - 2)$

(d) $m^2 + 4m - 21$

Answer: $(m - 3)(m + 7)$

6 These are perfect squares. Factorise each one and write your answer as a squared bracket.

(a) $x^2 + 6x + 9$

Answer: $(x + 3)^2$

(b) $x^2 - 10x + 25$

Answer: $(x - 5)^2$

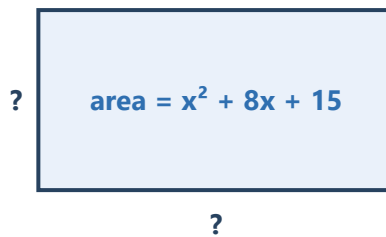
(c) $x^2 + 14x + 49$

Answer: $(x + 7)^2$

(d) $y^2 - 4y + 4$

Answer: $(y - 2)^2$

- 7 A rectangle has area $(x^2 + 8x + 15) \text{ cm}^2$. (a) Factorise the expression to find the lengths of its two sides. (b) Write a simplified expression for the perimeter.



- (a) the lengths of the two sides

Answer: $(x + 3) \text{ cm}$ and $(x + 5) \text{ cm}$

- (b) the perimeter

Answer: $4x + 16 \text{ cm}$

- 8 The area of a rectangular patio is $(x^2 + 7x + 12) \text{ m}^2$. Its width is $(x + 3) \text{ m}$. Find an expression for its length.

Answer: $(x + 4) \text{ m}$

- 9 Zak factorises $x^2 + 5x + 6$ as $(x + 1)(x + 6)$. Show that Zak is wrong, and find the correct answer.

Answer: $(x + 2)(x + 3)$

- 10 Explain why $x^2 + 5x + 7$ cannot be factorised into brackets with whole numbers.

Answer: No two whole numbers multiply to 7 and add to 5 (the pairs 1 and 7 add to 8).

- 11 Show that $x^2 - x - 6 = (x - 3)(x + 2)$ by expanding the brackets.

Answer: $x^2 - x - 6$

- 12 Factorise each expression.

(a) $x^2 - x - 72$

Answer: $(x - 9)(x + 8)$

(b) $x^2 + 14x + 48$

Answer: $(x + 6)(x + 8)$

(c) $x^2 - 17x + 72$

Answer: $(x - 9)(x - 8)$

(d) $x^2 + x - 90$

Answer: $(x - 9)(x + 10)$

13 Factorise each expression fully.

(a) $2x^2 + 10x + 12$

Answer: $2(x + 2)(x + 3)$

(b) $3x^2 - 12x - 15$

Answer: $3(x - 5)(x + 1)$

(c) $5x^2 - 5x - 30$

Answer: $5(x - 3)(x + 2)$

(d) $4x^2 - 4x - 24$

Answer: $4(x - 3)(x + 2)$

14 $x^2 + bx + 12$ can be factorised into two brackets with whole numbers. Find all the possible values of b .

Answer: $b = 7, 8, 13, -7, -8$ or -13

15 $x^2 + bx - 12$ can be factorised into two brackets with whole numbers. Find all the possible values of b .

Answer: $b = 11, 4, 1, -1, -4$ or -11

Exercise 7H Difference of two squares and mixed factorising

Non-calculator

1 Factorise each expression.

(a) $x^2 - 9$

Answer: $(x - 3)(x + 3)$

(b) $x^2 - 25$

Answer: $(x - 5)(x + 5)$

(c) $y^2 - 49$

Answer: $(y - 7)(y + 7)$

(d) $a^2 - 100$

Answer: $(a - 10)(a + 10)$

(e) $p^2 - 1$

Answer: $(p - 1)(p + 1)$

(f) $x^2 - 64$

Answer: $(x - 8)(x + 8)$

2 Factorise each expression.

(a) $4x^2 - 9$

Answer: $(2x - 3)(2x + 3)$

(b) $9y^2 - 1$

Answer: $(3y - 1)(3y + 1)$

(c) $16a^2 - 25$

Answer: $(4a - 5)(4a + 5)$

(d) $100x^2 - 81$

Answer: $(10x - 9)(10x + 9)$

3 Use the difference of two squares to work out each answer without a calculator.

(a) $51^2 - 49^2$

Answer: 200

(b) $99^2 - 1^2$

Answer: 9800

(c) $7.5^2 - 2.5^2$

Answer: 50

(d) $1001^2 - 999^2$

Answer: 4000

4 Factorise each expression fully. (Look for a common factor first.)

(a) $2x^2 - 8$

Answer: $2(x - 2)(x + 2)$

(b) $3x^2 - 75$

Answer: $3(x - 5)(x + 5)$

(c) $5y^2 - 20$

Answer: $5(y - 2)(y + 2)$

(d) $4a^2 - 100$

Answer: $4(a - 5)(a + 5)$

(e) $7x^2 - 7$

Answer: $7(x - 1)(x + 1)$

5 Factorise each expression.

(a) $x^2 - y^2$

Answer: $(x - y)(x + y)$

(b) $9a^2 - 4b^2$

Answer: $(3a - 2b)(3a + 2b)$

(c) $25p^2 - 16q^2$

Answer: $(5p - 4q)(5p + 4q)$

(d) $49m^2 - n^2$

Answer: $(7m - n)(7m + n)$

6 Factorise each expression.

(a) $25 - x^2$

Answer: $(5 - x)(5 + x)$

(b) $49 - 4y^2$

Answer: $(7 - 2y)(7 + 2y)$

(c) $1 - 9a^2$

Answer: $(1 - 3a)(1 + 3a)$

7 A square of side 10 cm has a small square of side x cm cut from one corner. (a) Write an expression for the area that is left. (b) Factorise your expression.

(a) the area left

Answer: $(100 - x^2) \text{ cm}^2$

(b) factorise it

Answer: $(10 - x)(10 + x)$

8 One square has side 13 cm and another has side 7 cm. Work out the difference between their areas without a calculator.

Answer: 120 cm^2

9 Explain why $x^2 + 16$ cannot be factorised using the difference of two squares.

Answer: There is a plus sign, not a minus sign. $(x + 4)(x - 4)$ gives $x^2 - 16$.

10 Ravi factorises $x^2 - 16$ as $(x - 4)(x - 4)$. Is he correct? Explain.

Answer: No. $(x - 4)(x - 4) = x^2 - 8x + 16$. The correct answer is $(x - 4)(x + 4)$.

11 Is $x^2 - 12$ a difference of two squares? Explain your answer.

Answer: No, because 12 is not a square number.

12 Factorise each expression fully. Each one uses a different method.

(a) $x^2 + 9x$

Answer: $x(x + 9)$

(b) $x^2 - x - 20$

Answer: $(x - 5)(x + 4)$

(c) $x^2 - 36$

Answer: $(x - 6)(x + 6)$

(d) $3x^2 + 6x$

Answer: $3x(x + 2)$

(e) $y^2 - 2y - 24$

Answer: $(y - 6)(y + 4)$

(f) $2x^2 - 50$

Answer: $2(x - 5)(x + 5)$

13 Factorise each expression.

(a) $16x^2 - 81y^2$

Answer: $(4x - 9y)(4x + 9y)$

(b) $121a^2 - 4b^2$

Answer: $(11a - 2b)(11a + 2b)$

(c) $36p^2q^2 - 1$

Answer: $(6pq - 1)(6pq + 1)$

(d) $25x^4 - 9$

Answer: $(5x^2 - 3)(5x^2 + 3)$

14 Factorise fully $x^4 - 16$.

Answer: $(x - 2)(x + 2)(x^2 + 4)$

15 Factorise $(x + 3)^2 - 16$.

Answer: $(x - 1)(x + 7)$

16 Work out $8.5^2 - 1.5^2$ without a calculator, and show your method.

Answer: 70