

GCSE Maths: Identities and algebraic proof (Higher): answers

GCSE Maths - Algebra Basics - Higher - Lesson 7.9

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at websink.site/gcse/maths/ch07-algebra-basics/higher/identities-and-algebraic-proof/

Exercise 7J Identities and proof Non-calculator

1 For each statement, say whether it is an equation or an identity. Give a reason.

(a) $3(x + 2) = 3x + 6$

Answer: Identity

(b) $2x + 5 = 11$

Answer: Equation

(c) $x^2 - 9 = (x - 3)(x + 3)$

Answer: Identity

(d) $4x = x + 12$

Answer: Equation

(e) $(x + 1)^2 = x^2 + 1$

Answer: Equation

(f) $5x - 2x = 3x$

Answer: Identity

2 Show that each identity is true.

(a) $2(x + 5) + 3 \equiv 2x + 13$

Answer: $2x + 13$

(b) $4(x + 3) - 2(x + 1) \equiv 2x + 10$

Answer: $2x + 10$

(c) $x(x + 3) - 3x \equiv x^2$

Answer: x^2

(d) $3(2x - 1) - 6x \equiv -3$

Answer: -3

3 Show that the sum of any four consecutive even numbers is a multiple of 4.

Answer: $4(2n + 3)$

4 Show that the sum of any two even numbers is even.

Answer: $2(a + b)$

5 Show that the product of an odd number and an even number is always even.

Answer: $2n(2m + 1)$

6 Chen says: "The sum of any two odd numbers is odd." Give a counter-example to show that Chen is wrong.

Answer: $3 + 5 = 8$, which is even

7 Beth says: "Think of a number. Double it. Add 6. Halve the answer. Subtract the number you first thought of. The answer is always 3." Use algebra to show that Beth is right.

Answer: 3

8 Think of a number. Add 4. Multiply the answer by 3. Subtract 12. Divide by 3. Show that you always end up with the number you started with.

Answer: n

9 n is an integer. Explain why $2n$ is always even and $2n + 1$ is always odd.

Answer: $2n$ is $2 \times$ an integer, so it is even. $2n + 1$ is one more than an even number, so it is odd.

10 Show that $(x + 3)^2 - 9 \equiv x(x + 6)$.

Answer: $x(x + 6)$

11 Mia says that $(x + 2)^2 \equiv x^2 + 4$ for all values of x . Is Mia right? Explain.

Answer: No. It is only true when $x = 0$, so it is an equation, not an identity.

12 Show that the sum of an odd number and an even number is always odd.

Answer: $2(m + n) + 1$

13 The identity $3(x + a) + 2(x - 1) \equiv bx + 7$ is true for every value of x . Find the values of a and b .

Answer: $a = 3$ and $b = 5$

14 Prove each statement.

(a) The sum of any five consecutive integers is a multiple of 5.

Answer: $5(n + 2)$

(b) The sum of any two consecutive odd numbers is a multiple of 4.

Answer: $4(n + 1)$

(c) The sum of the squares of any two consecutive integers is odd.

Answer: $2(n^2 + n) + 1$

(d) The product of any two odd numbers is odd.

Answer: $2(2mn + m + n) + 1$

15 Prove or show each statement.

(a) $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 12, for any integer n .

Answer: $12n$

(b) $(a + b)^2 - (a - b)^2 \equiv 4ab$

Answer: $4ab$

(c) $(x + 3)(x + 5) + 1$ is a square number (or a squared expression).

Answer: $(x + 4)^2$

(d) The sum of any four consecutive integers is never a multiple of 4.

Answer: $4(n + 1) + 2$

16 Each statement is false. Find a counter-example for each one.

(a) $n^2 - n + 11$ is prime for every positive integer n .

Answer: $n = 11$: $121 = 11 \times 11$

(b) x^2 is always bigger than x .

Answer: $x = 1$: $1^2 = 1$, which is not bigger than 1 (also $x = \frac{1}{2}$: $\frac{1}{4} < \frac{1}{2}$)

(c) The sum of two square numbers is never a square number.

Answer: $9 + 16 = 25$

17 Prove that the product of any two consecutive integers is even.

Answer: $n(n + 1)$ always has a factor of 2

18 Prove that $(2n + 1)^2 - 1$ is a multiple of 8 for every integer n . (You may use the fact that the product of two consecutive integers is even.)

Answer: $8k$

19 Prove that $(n + 3)^2 - (n - 2)^2$ is always an odd multiple of 5.

Answer: $5(2n + 1)$