

GCSE Maths: Midpoints and lengths of line segments (Higher): answers

GCSE Maths - Straight-Line Graphs and Coordinates - Higher - Lesson 12.2

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at websink.site/gcse/maths/ch12-straight-line-graphs-coordinates/higher/midpoints-and-lengths-of-line-segments/

Exercise 12B Midpoints Non-calculator

1 Find the midpoint of each line segment.

(a) A(2, 4), B(8, 10)

Answer: (5, 7)

(b) A(0, 6), B(10, 2)

Answer: (5, 4)

(c) A(1, 3), B(9, 7)

Answer: (5, 5)

(d) A(4, 2), B(12, 8)

Answer: (8, 5)

2 Find the midpoint of each line segment. Take care with the negative numbers.

(a) A(-4, 2), B(6, 8)

Answer: (1, 5)

(b) A(-7, -3), B(1, 5)

Answer: (-3, 1)

(c) A(-2, 5), B(4, -1)

Answer: (1, 2)

(d) A(-8, -6), B(-2, -4)

Answer: (-5, -5)

3 Find the midpoint of each line segment. The coordinates of the midpoint may be decimals.

(a) A(1, 2), B(6, 5)

Answer: (3.5, 3.5)

(b) A(-3, 4), B(2, -1)

Answer: (-0.5, 1.5)

(c) A(0, -5), B(7, 0)

Answer: (3.5, -2.5)

(d) $A(-1, 3)$, $B(0, 6)$

Answer: $(-0.5, 4.5)$

- 4 Aisha lives at $A(-6, 2)$ and Ben lives at $B(4, 8)$ on a map where each unit is 1 km. They agree to meet at the point exactly halfway between their homes. Write the coordinates of the meeting point.

Answer: $(-1, 5)$

- 5 AB is a diameter of a circle. A is $(-5, 2)$ and B is $(3, -4)$. Find the coordinates of the centre of the circle.

Answer: $(-1, -1)$

- 6 Tom says the midpoint of $(2, 6)$ and $(8, -2)$ is $(3, -4)$. Find his mistake and work out the correct midpoint.

Answer: Tom subtracted; the midpoint is $(5, 2)$

- 7 A is $(-4, 6)$ and B is $(8, -2)$. C is the midpoint of AB. D is the midpoint of AC. Find the coordinates of D.

Answer: $(-1, 4)$

- 8 ABCD is a rectangle with $A(-1, -2)$, $B(5, -2)$, $C(5, 4)$ and $D(-1, 4)$.

(a) Find the midpoint of the diagonal AC.

Answer: $(2, 1)$

(b) Find the midpoint of the diagonal BD.

Answer: $(2, 1)$

(c) What do your answers show about the diagonals of a rectangle?

Answer: They have the same midpoint: the diagonals bisect each other.

- 9 A path joins a campsite at $C(2, -3)$ to a lake at $L(10, 5)$ on a map where each unit is 100 m. A bench B is halfway between the campsite and the lake. A second bench S is halfway between the first bench and the lake. Find the coordinates of B and S.

(a) the first bench B

Answer: $(6, 1)$

(b) the second bench S

Answer: $(8, 3)$

- 10 M is the midpoint of AB. Find the coordinates of B in each case.

(a) $A(1, 1)$, $M(4, 3)$

Answer: $(7, 5)$

(b) $A(-2, 5)$, $M(1, 2)$

Answer: $(4, -1)$

(c) $A(6, -3)$, $M(2, 0)$

Answer: $(-2, 3)$

- 11 ABCD is a parallelogram. A is $(-2, 1)$, B is $(3, -1)$ and C is $(6, 5)$. Use the midpoint of the diagonals to find the coordinates of D.

Answer: $(1, 7)$

- 12** (a) Work out the midpoint of (3, 5) and (8, 2). (b) Work out the midpoint of (3, 5) and (9, 3). (c) Explain when the midpoint of two points with whole-number coordinates also has whole-number coordinates.

Answer: (a) (5.5, 3.5) (b) (6, 4) (c) when both coordinate sums are even

- 13** Find the midpoint of the points $(2a, 3b)$ and $(4a, b)$. Give each coordinate in its simplest form.

Answer: $(3a, 2b)$

Exercise 12C Lengths of line segments Calculator allowed

- 1** Work out the exact length of each line segment. Each answer is a whole number.

- (a) A(0, 0), B(3, 4)

Answer: 5

- (b) A(1, 1), B(6, 13)

Answer: 13

- (c) A(-2, -1), B(4, 7)

Answer: 10

- (d) A(-3, 2), B(2, 14)

Answer: 13

- 2** Work out the length of each line segment. Give your answers correct to 1 decimal place.

- (a) A(-1, 3), B(4, 5)

Answer: 5.4

- (b) A(-3, -2), B(2, 4)

Answer: 7.8

- (c) A(2, -3), B(7, 5)

Answer: 9.4

- (d) A(-5, 1), B(-1, -6)

Answer: 8.1

- 3** A map uses a grid where each unit is 1 km. A harbour H is at (0, 0), a lighthouse L is at (5, 12) and a buoy B is at (-4, 0). Find (a) the distance HL, (b) the distance LB, (c) the total distance of a trip from H to L and then to B.

- (a) the distance HL

Answer: 13 km

- (b) the distance LB

Answer: 15 km

- (c) the total distance H to L to B

Answer: 28 km

- 4** The vertices of triangle ABC are A(1, 1), B(7, 1) and C(7, 9). Work out the perimeter of the triangle.

Answer: 24

- 5 Which of the points $(3, -5)$ and $(-4, 4)$ is further from the origin? Show your working.

Answer: $(3, -5)$

- 6 A drone flies in a straight line from the point $(-2, 3)$ to the point $(10, 8)$ on a map. Each unit on the map is 100 m. How far does it fly?

Answer: 1300 m

- 7 Two towns are at $A(-6, -2)$ and $B(6, 3)$ on a map where each unit is 1 km.

- (a) How far apart are the towns?

Answer: 13 km

- (b) A petrol station is halfway between the towns. Find its coordinates.

Answer: $(0, 0.5)$

- 8 Find the exact length of each line segment. Give your answers as whole numbers or simplified surds.

- (a) $A(-2, 1)$, $B(4, 5)$

Answer: $2\sqrt{13}$

- (b) $A(1, -3)$, $B(4, 4)$

Answer: $\sqrt{58}$

- (c) $A(0, 0)$, $B(-6, -6)$

Answer: $6\sqrt{2}$

- (d) $A(5, 2)$, $B(-1, -6)$

Answer: 10

- 9 The vertices of triangle ABC are $A(1, 1)$, $B(5, 3)$ and $C(3, 7)$.

- (a) Find the exact length of AB.

Answer: $2\sqrt{5}$

- (b) Find the exact length of BC.

Answer: $2\sqrt{5}$

- (c) Find the exact length of AC.

Answer: $2\sqrt{10}$

- (d) Show that ABC is an isosceles triangle, and that it is right-angled.

Answer: $AB = BC$ (isosceles) and $AB^2 + BC^2 = AC^2$ (right angle at B)

- 10 AB is a diameter of a circle with $A(-3, 1)$ and $B(5, 7)$.

- (a) Find the coordinates of the centre of the circle.

Answer: $(1, 4)$

(b) Work out the length of the diameter AB.

Answer: 10

(c) Write down the radius of the circle.

Answer: 5

(d) Does the point P(1, 9) lie on the circle? Show how you decide.

Answer: Yes

11 Mia says the distance from (1, 2) to (4, 6) is 7, because $3 + 4 = 7$. Explain why she is wrong and find the correct distance.

Answer: Correct distance is 5

12 The point P(k , 0) is 5 units from the point Q(0, 3). Find the two possible values of k .

Answer: $k = 4$ or $k = -4$

13 A boat sails in a straight line from (−4, −3) to (4, 3) on a map where each unit is 1 km. How far does the boat sail?

Answer: 10 km