

GCSE Maths: Sketching quadratics and completing the square (Higher): answers

GCSE Maths - Quadratic and Other Graphs - Higher - Lesson 13.9

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at websink.site/gcse/maths/ch13-quadratic-other-graphs/higher/sketching-quadratics-and-completing-the-square/

Exercise 13I Sketching quadratics and completing the square Non-calculator

1 Write each expression in the form $(x + p)^2 + q$.

(a) $x^2 + 4x + 1$

Answer: $y = (x + 2)^2 - 3$

(b) $x^2 - 6x + 2$

Answer: $y = (x - 3)^2 - 7$

(c) $x^2 + 10x$

Answer: $y = (x + 5)^2 - 25$

(d) $x^2 - 2x - 5$

Answer: $y = (x - 1)^2 - 6$

(e) $x^2 + 3x$

Answer: $y = (x + 1.5)^2 - 2.25$

(f) $x^2 - 8x + 20$

Answer: $y = (x - 4)^2 + 4$

2 Complete the square to find the coordinates of the turning point of each graph.

(a) $y = x^2 - 4x + 7$

Answer: (2, 3)

(b) $y = x^2 + 6x + 5$

Answer: (-3, -4)

(c) $y = x^2 - 10x + 30$

Answer: (5, 5)

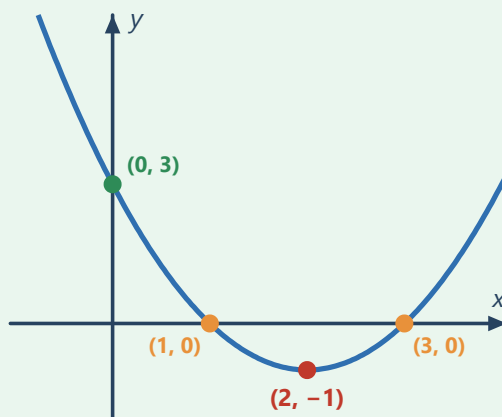
(d) $y = x^2 + x + 1$

Answer: (-0.5, 0.75)

3 Sketch each graph. Mark the coordinates of the y-intercept, any roots and the turning point.

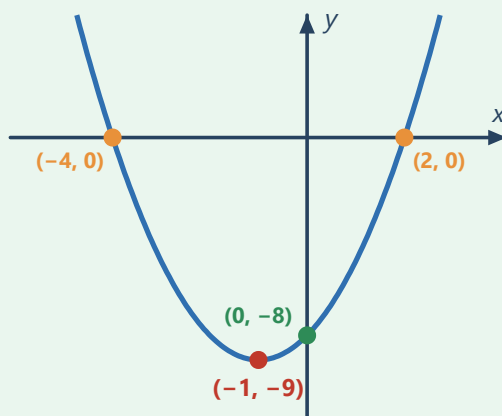
(a) Sketch $y = x^2 - 4x + 3$

Answer: U-shaped; y-intercept (0, 3); roots (1, 0) and (3, 0); turning point (2, -1).



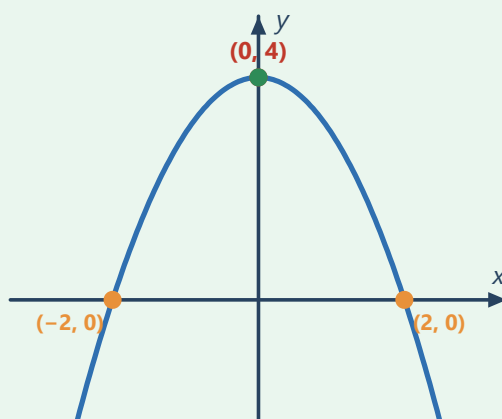
(b) Sketch $y = x^2 + 2x - 8$

Answer: U-shaped; y-intercept (0, -8); roots (-4, 0) and (2, 0); turning point (-1, -9).



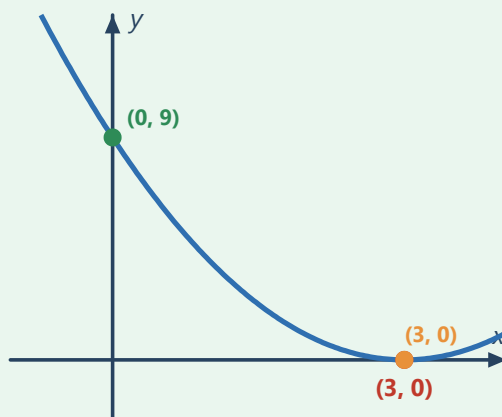
(c) Sketch $y = -x^2 + 4$

Answer: n-shaped; y-intercept (0, 4); roots (-2, 0) and (2, 0); turning point (0, 4).



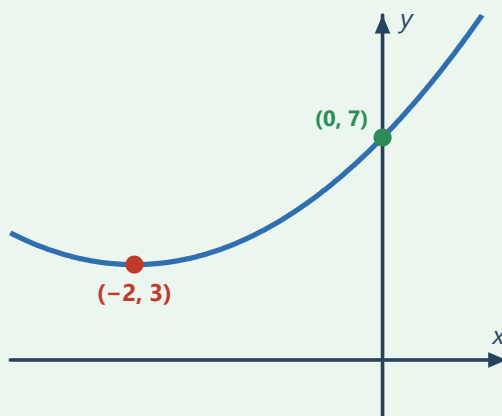
(d) Sketch $y = x^2 - 6x + 9$

Answer: U-shaped; y-intercept (0, 9); one root at (3, 0); turning point (3, 0).



(e) Sketch $y = x^2 + 4x + 7$

Answer: U-shaped; y-intercept (0, 7); no roots; turning point (-2, 3).



4 Solve each equation by completing the square. Give your answers as surds.

(a) $x^2 - 4x + 1 = 0$

Answer: $x = 2 \pm \sqrt{3}$

(b) $x^2 + 6x + 2 = 0$

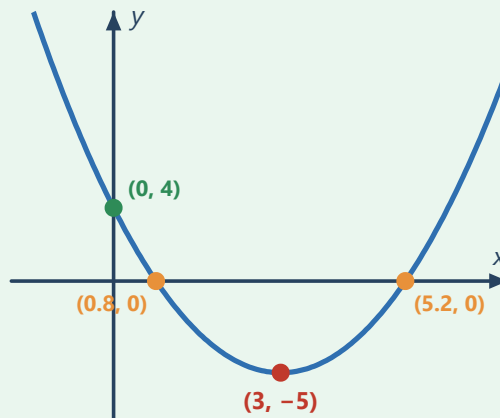
Answer: $x = -3 \pm \sqrt{7}$

(c) $x^2 - 2x - 5 = 0$

Answer: $x = 1 \pm \sqrt{6}$

5 Sketch the graph of $y = x^2 - 6x + 4$, showing the exact coordinates of the points where it crosses the x-axis.

Answer: U-shaped; y-intercept (0, 4); roots $(3 + \sqrt{5}, 0)$ and $(3 - \sqrt{5}, 0)$; turning point (3, -5).



- 6 Write down the coordinates of the turning point of each graph, and say whether it is a maximum or a minimum. (No working out is needed.)

(a) $y = (x - 5)^2 + 3$

Answer: (5, 3) (minimum)

(b) $y = (x + 2)^2 - 7$

Answer: (-2, -7) (minimum)

(c) $y = -(x - 1)^2 + 4$

Answer: (1, 4) (maximum)

- 7 Show that $x^2 - 8x + 19$ is always greater than 0.

Answer: $(x - 4)^2 + 3 \geq 3 > 0$

- 8 The curve $y = x^2 + bx + c$ has its turning point at (3, -2). Find the values of b and c .

Answer: $b = -6$, $c = 7$

- 9 Write $2x^2 - 12x + 7$ in the form $a(x + p)^2 + q$. Write down the minimum value of $2x^2 - 12x + 7$ and the value of x when it happens.

Answer: (3, -11) (minimum value -11 at $x = 3$)

- 10 By completing the square, find the coordinates of the maximum point of $y = -x^2 + 6x - 5$.

Answer: (3, 4)

- 11 The path of a ball has equation $y = 20x - 5x^2$. Complete the square to find the greatest height and where it happens.

Answer: (2, 20) (greatest height 20)

- 12 The graph of $y = x^2 + kx + 9$ touches the x -axis at exactly one point, and k is positive. Find k and the coordinates of the point.

Answer: $k = 6$, point (-3, 0)

- 13** Priya says that the turning point of $y = (x + 4)^2 - 5$ is $(4, -5)$. Explain her mistake and give the correct coordinates.

Answer: $(-4, -5)$

- 14** Prove that the turning point of $y = x^2 + bx + c$ is at $x = -b/2$, for any numbers b and c .

Answer: $(x + b/2)^2 + c - b^2/4$ has its minimum when $x = -b/2$