

GCSE Maths: Completing the square (Higher): answers

GCSE Maths - Quadratic and Simultaneous Equations - Higher - Lesson 14.10

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at websink.site/gcse/maths/ch14-quadratics-simultaneous/higher/completing-the-square/

Exercise 14J Completing the square Non-calculator

1 Write each expression in the form $(x + p)^2 + q$.

(a) $x^2 + 4x + 1$

Answer: $(x + 2)^2 - 3$

(b) $x^2 + 10x + 3$

Answer: $(x + 5)^2 - 22$

(c) $x^2 - 6x + 2$

Answer: $(x - 3)^2 - 7$

(d) $x^2 - 2x - 5$

Answer: $(x - 1)^2 - 6$

(e) $x^2 + 3x + 1$

Answer: $(x + \frac{3}{2})^2 - \frac{5}{4}$

(f) $x^2 - 12x + 40$

Answer: $(x - 6)^2 + 4$

2 Write each expression in the form $a(x + p)^2 + q$.

(a) $2x^2 + 8x + 3$

Answer: $2(x + 2)^2 - 5$

(b) $3x^2 - 12x + 5$

Answer: $3(x - 2)^2 - 7$

(c) $2x^2 - 4x - 1$

Answer: $2(x - 1)^2 - 3$

(d) $5x^2 + 10x + 2$

Answer: $5(x + 1)^2 - 3$

(e) $4x^2 + 8x + 9$

Answer: $4(x + 1)^2 + 5$

(f) $-x^2 + 6x + 1$

Answer: $-(x - 3)^2 + 10$

3 Solve by completing the square.

$x^2 + 4x - 1 = 0$. Give your answers in the form $p \pm \sqrt{q}$.

Answer: $x = -2 \pm \sqrt{5}$

4 Solve by completing the square.

$x^2 - 6x + 1 = 0$. Give your answers in the form $p \pm \sqrt{q}$.

Answer: $x = 3 \pm 2\sqrt{2}$

5 Solve by completing the square.

$x^2 + 8x + 3 = 0$. Give your answers in the form $p \pm \sqrt{q}$.

Answer: $x = -4 \pm \sqrt{13}$

6 Solve by completing the square.

$x^2 - 2x - 5 = 0$. Give your answers in the form $p \pm \sqrt{q}$.

Answer: $x = 1 \pm \sqrt{6}$

7 Solve by completing the square.

$x^2 + 6x + 2 = 0$. Give your answers correct to 2 decimal places.

Answer: $x = -5.65$ or $x = -0.35$

8 Solve by completing the square.

$2x^2 + 12x + 6 = 0$. Give your answers in the form $p \pm \sqrt{q}$.

Answer: $x = -3 \pm \sqrt{6}$

9 Solve by completing the square (the numbers are all multiples of 3).

$3x^2 - 18x + 15 = 0$. Give your answers in the form $p \pm \sqrt{q}$.

Answer: $x = 1$ or $x = 5$

10 Write each quadratic in completed-square form and write down the coordinates of its turning point.

(a) $y = x^2 + 6x + 5$

Answer: $(-3, -4)$

(b) $y = x^2 - 4x + 7$

Answer: $(2, 3)$

(c) $y = x^2 + 2x - 8$

Answer: $(-1, -9)$

(d) $y = 2x^2 - 8x + 9$

Answer: $(2, 1)$

(e) $y = x^2 - 10x$

Answer: $(5, -25)$

(f) $y = -x^2 + 6x + 1$

Answer: (3, 10)

- 11** Find the minimum value of $x^2 - 6x + 15$ and the value of x at which it occurs.

Answer: Minimum value 6, when $x = 3$

- 12** Show that $x^2 + 6x + 11$ is always positive, whatever the value of x .

Answer: $= (x + 3)^2 + 2$, and a square is never negative, so it is at least 2

- 13** $x^2 - 10x + 30 = (x + p)^2 + q$ for all values of x . Find the values of p and q .

Answer: $p = -5$, $q = 5$

- 14** The curve $y = x^2 + bx + c$ has its turning point at (3, -2). Find the values of b and c .

Answer: $b = -6$, $c = 7$

- 15** Complete the square to explain why $x^2 + 6x + 11 = 0$ has no real solutions.

Answer: Minimum value is 2 (at $x = -3$), so the expression is never 0