

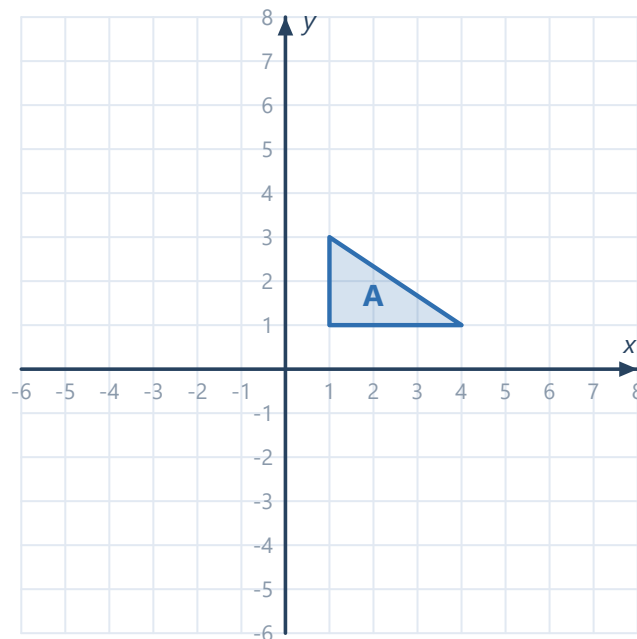
GCSE Maths: Combinations of transformations and invariant points (Higher): answers

GCSE Maths - Transformations and Similarity - Higher - Lesson 21.6

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at websink.site/gcse/maths/ch21-transformations-congruence-similarity/higher/combinations-of-transformations-and-invariant-points/

Exercise 21F Combinations and invariant points Non-calculator

- 1 Shape A has vertices at (1, 1), (4, 1), (1, 3). For each combination, work out the coordinates of the vertices of the final image.



- (a) Reflect in the x -axis, then reflect in the y -axis.

Answer: C: (−1, −1), (−4, −1), (−1, −3)

- (b) Translate by $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$, then reflect in the line $y = x$.

Answer: C: (4, 3), (4, 6), (6, 3)

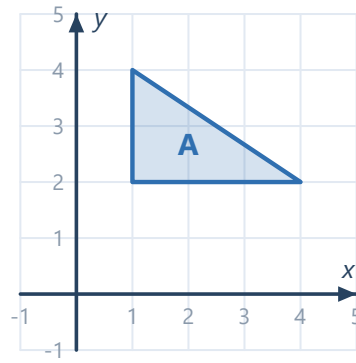
- (c) Rotate 90° anticlockwise about the origin, then translate by $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$.

Answer: C: (−2, 5), (−2, 8), (−4, 5)

- (d) Rotate 180° about the origin, then reflect in the line $x = 1$.

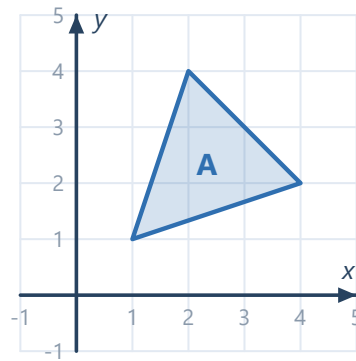
Answer: C: (3, −1), (6, −1), (3, −3)

- 2 A is mapped to B by a reflection in the line $y = 0$. B is mapped to C by a reflection in the line $y = 3$. Describe fully the single transformation that maps A onto C. A has vertices at (1, 2), (4, 2), (1, 4).



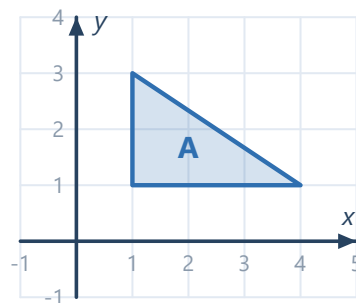
Answer: Translation by $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$

- 3 A is mapped to B by a reflection in the line $y = 0$. B is mapped to C by a reflection in the line $y = x$. Describe fully the single transformation that maps A onto C. A has vertices at (1, 1), (4, 2), (2, 4).



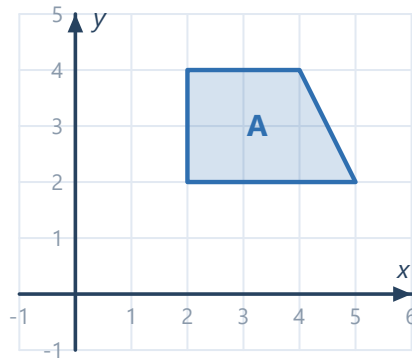
Answer: Rotation, 90° anticlockwise, centre (0, 0)

- 4 A is mapped to B by a rotation of 90° clockwise about (0, 0). B is mapped to C by a rotation of 180° about (0, 0). Describe fully the single transformation that maps A onto C. A has vertices at (1, 1), (4, 1), (1, 3).



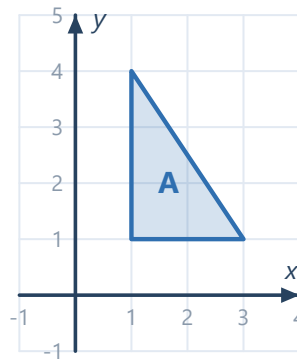
Answer: Rotation, 90° anticlockwise, centre (0, 0)

- 5 A is mapped to B by a reflection in the line $x = 1$. B is mapped to C by a reflection in the line $y = 1$. Describe fully the single transformation that maps A onto C. A has vertices at (2, 2), (5, 2), (4, 4), (2, 4).



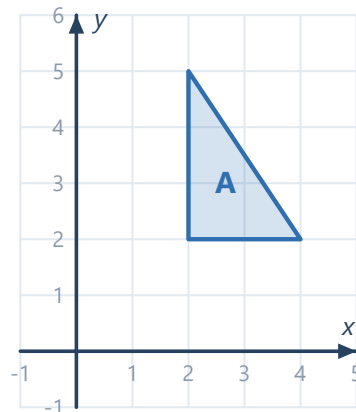
Answer: Rotation, 180° , centre (1, 1)

- 6 A is mapped to B by a translation by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$. B is mapped to C by a reflection in the line $x = 0$. Describe fully the single transformation that maps A onto C. A has vertices at (1, 1), (3, 1), (1, 4).



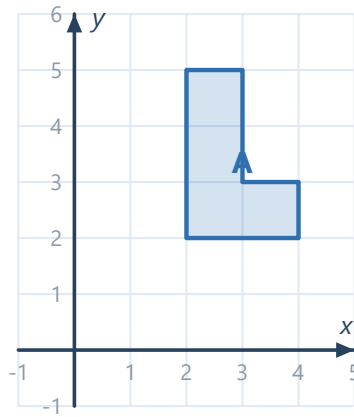
Answer: Reflection in the line $x = -1$

- 7 A is mapped to B by an enlargement with scale factor 2 and centre (0, 0). B is mapped to C by a translation by $\begin{pmatrix} -3 \\ -3 \end{pmatrix}$. Describe fully the single transformation that maps A onto C. A has vertices at (2, 2), (4, 2), (2, 5).



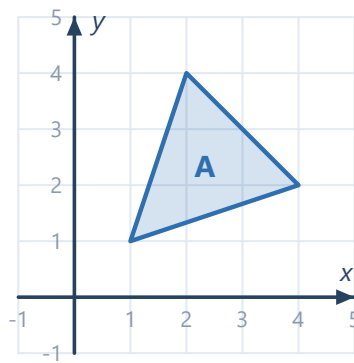
Answer: Enlargement, scale factor 2, centre (3, 3)

- 8 Shape A has vertices at (2, 2), (4, 2), (4, 3), (3, 3), (3, 5), (2, 5). Shape B is found by doing a reflection in the line $y = x$ and then a translation by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$. Shape C is found by doing the translation by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ first and then the reflection in the line $y = x$. Is shape C in the same place as shape B? Show your working.



Answer: No. B: (5, 2), (5, 4), (6, 4), (6, 3), (8, 3), (8, 2); C: (2, 5), (2, 7), (3, 7), (3, 6), (5, 6), (5, 5)

- 9 Shape A has vertices at (1, 1), (4, 2), (2, 4). Shape B is found by doing a rotation of 90° anticlockwise about (1, 1) and then a reflection in the line $x = 0$. Shape C is found by doing the reflection in the line $x = 0$ first and then the rotation of 90° anticlockwise about (1, 1). Is shape C in the same place as shape B? Show your working.



Answer: No. B: (-1, 1), (0, 4), (2, 2); C: (1, -1), (0, -4), (-2, -2)

- 10 Find the invariant point(s) of each combination of transformations. (Use the single transformation that does the same job.)

- (a) Reflection in $x = 1$, then reflection in $x = 4$

Answer: None (every point moves).

- (b) Reflection in the x -axis, then reflection in the y -axis

Answer: Only the centre, (0, 0).

- (c) Rotation 90° clockwise about (1, 1), twice

Answer: Only the centre, (1, 1).

- (d) Enlargement scale factor 2 about the origin, then translation by $\begin{pmatrix} -3 \\ -3 \end{pmatrix}$

Answer: Only the centre, (3, 3).

- 11 Reflection in $x = p$ followed by reflection in $x = q$ is a translation. Show that the translation is $\begin{pmatrix} 2(q - p) \\ 0 \end{pmatrix}$.

Answer: The x -coordinate increases by $2(q - p)$

- 12** Show that reflection in the x -axis followed by reflection in the y -axis is a rotation of 180° about the origin, using the point (a, b) .

Answer: $(a, b) \rightarrow (a, -b) \rightarrow (-a, -b)$

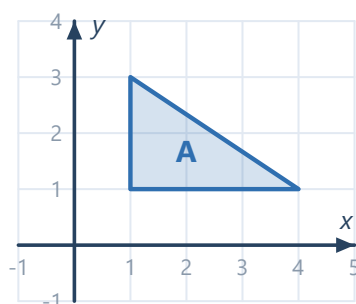
- 13** Jo says: "When you do two transformations, the order never matters." Give a counter-example to show that Jo is wrong. Use the point $(2, 1)$, a reflection in the x -axis and a translation by $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

Answer: Reflect then translate: $(5, 1)$. Translate then reflect: $(5, -3)$.

- 14** Reflection in $y = x$ followed by reflection in $y = -x$ is a single transformation. Describe it fully. Use the point (a, b) to show your working.

Answer: Rotation, 180° , centre $(0, 0)$

- 15** A is mapped to B by a reflection in the line $y = 0$. B is mapped to C by a reflection in the line $y = x$. Describe fully the single transformation that maps A onto C. A has vertices at $(1, 1)$, $(4, 1)$, $(1, 3)$.



Answer: Rotation, 90° anticlockwise, centre $(0, 0)$

- 16 Challenge.** Shape A is reflected in the line $x = 2$ to give B. Shape B is then reflected in the line $y = 1$ to give C. Describe fully the single transformation that maps A onto C, and find its centre. Use the point $(5, 3)$ to help.

Answer: Rotation, 180° , centre $(2, 1)$