

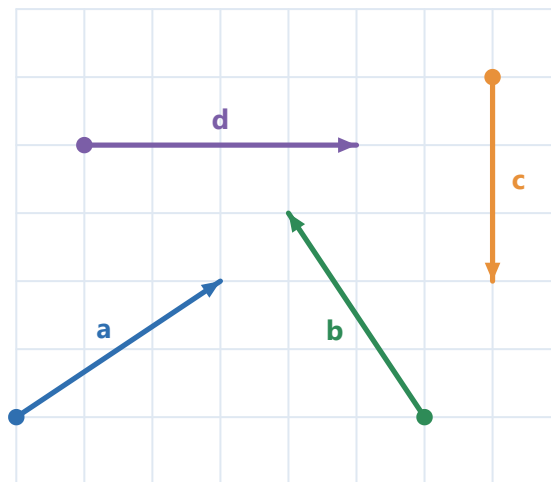
GCSE Maths: Vectors: column vectors, addition and scalar multiples (Foundation): answers

GCSE Maths - Loci, Bearings and Vectors - Foundation - Lesson 22.8

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at websink.site/gcse/maths/ch22-constructions-loci-bearings-vectors/foundation/vectors-column-vectors-addition-and-scalar-multiples/

Exercise 22J Column vectors and translations Non-calculator

1 Write down each vector as a column vector.



(a) a

Answer: $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

(b) b

Answer: $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

(c) c

Answer: $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

(d) d

Answer: $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

2 Write each movement as a column vector.

(a) 3 right and 2 up

Answer: $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

- (b) 4 left and 1 up

Answer: $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$

- (c) 2 right and 5 down

Answer: $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$

- (d) 6 left

Answer: $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$

3 Work out the column vector from the first point to the second point.

- (a) A(1, 2) to B(4, 6)

Answer: $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

- (b) A(-2, 3) to B(2, -1)

Answer: $\begin{pmatrix} 4 \\ -4 \end{pmatrix}$

- (c) A(5, 4) to B(1, 4)

Answer: $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$

- (d) A(0, -3) to B(-4, -6)

Answer: $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$

4 Work out the coordinates of the end point B.

- (a) A(2, 1), vector from A to B is $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

Answer: (5, 5)

- (b) A(-1, 5), vector from A to B is $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$

Answer: (3, 3)

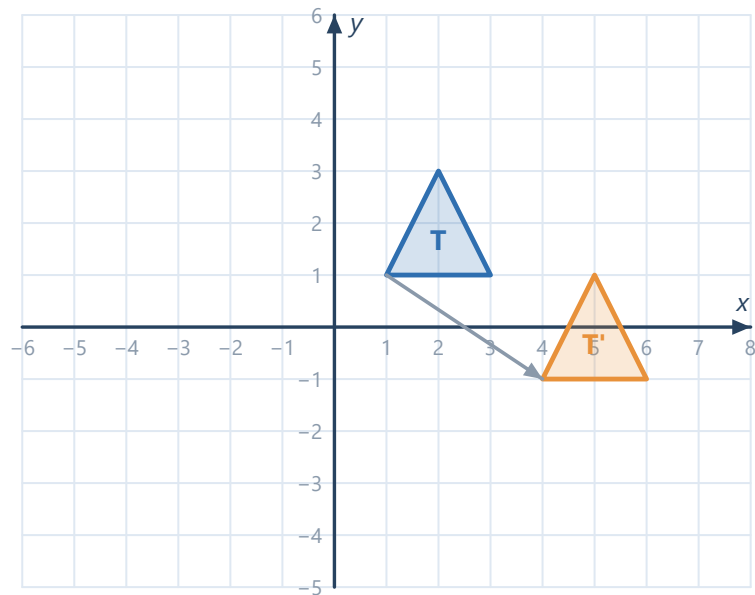
- (c) A(0, 0), vector from A to B is $\begin{pmatrix} -3 \\ -6 \end{pmatrix}$

Answer: (-3, -6)

- (d) A(6, -2), vector from A to B is $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$

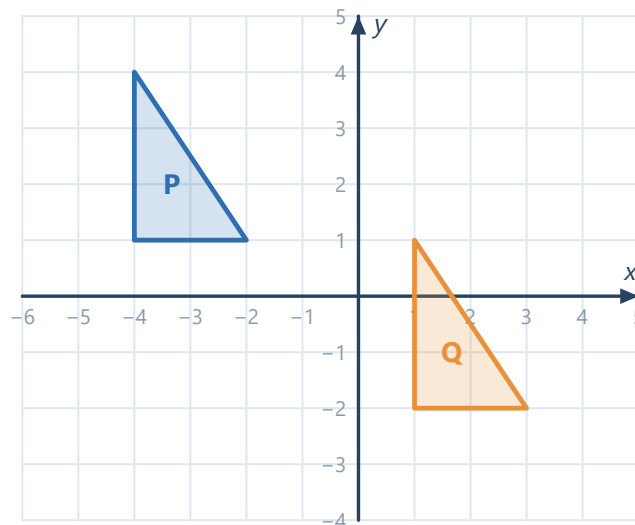
Answer: (4, 3)

5 Triangle T has vertices A(1, 1), B(3, 1) and C(2, 3). It is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ to give T'. Write the coordinates of A', B' and C'.



Answer: $A' (4, -1)$, $B' (6, -1)$, $C' (5, 1)$

- 6** Shape P is translated to shape Q. Describe the translation using a column vector.



Answer: $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$

- 7** $\mathbf{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$. Write down each vector.

(a) $-\mathbf{a}$

Answer: $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

(b) $2\mathbf{a}$

Answer: $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$

(c) $-3a$

Answer: $\begin{pmatrix} -9 \\ 6 \end{pmatrix}$

- 8 A drone moves $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$, then $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$, then $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$ (in metres). Work out its total movement as a column vector and its distance from where it started.

Answer: $\begin{pmatrix} 7 \\ 0 \end{pmatrix}$, 7 m

- 9 ABCD is a parallelogram with A at (1, 1). The vector from A to B is $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and the vector from A to D is $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$. Write down (a) the vector from B to C, (b) the vector from D to C, (c) the coordinates of C.

Answer: (a) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$, (b) $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$, (c) (6, 5)

- 10 Are the vectors $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ equal? Give a reason.

Answer: No. They have different numbers, so different directions.

- 11 Triangle T is translated by $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$, and then the image is translated by $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$. **Show that** the final triangle is in the same position as T.

Answer: $\begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

- 12 A translation maps the point A(2, 3) to A'(7, 1). B is the point (5, -4). Work out the coordinates of B', the image of B under the same translation.

Answer: (10, -6)

- 13 A ship sails 6 km east and then 8 km north. Write its movement as a column vector and find how far it is from its start.

Answer: $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$, 10 km

- 14 **Challenge.** A translation moves the point P(1, 2) to Q and then Q is moved by a second translation $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ to R(4, 1). Work out the coordinates of Q and the vector from P to Q.

Answer: Q(7, -4); $\begin{pmatrix} 6 \\ -6 \end{pmatrix}$

Exercise 22K Combining vectors: addition, scalar multiples and magnitude

Calculator allowed

- 1 Work out each sum.

(a) $\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

Answer: $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$

(b) $\begin{pmatrix} -1 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

Answer: $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(c) $\begin{pmatrix} 6 \\ -4 \end{pmatrix} + \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

Answer: $\begin{pmatrix} 4 \\ -6 \end{pmatrix}$

(d) $\begin{pmatrix} 0 \\ 7 \end{pmatrix} + \begin{pmatrix} -5 \\ 3 \end{pmatrix}$

Answer: $\begin{pmatrix} -5 \\ 10 \end{pmatrix}$

2 Work out each difference.

(a) $\begin{pmatrix} 5 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Answer: $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(b) $\begin{pmatrix} 1 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

Answer: $\begin{pmatrix} -3 \\ -7 \end{pmatrix}$

(c) $\begin{pmatrix} -3 \\ 6 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

Answer: $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$

(d) $\begin{pmatrix} 2 \\ -4 \end{pmatrix} - \begin{pmatrix} -6 \\ -9 \end{pmatrix}$

Answer: $\begin{pmatrix} 8 \\ 5 \end{pmatrix}$

3 Work out each scalar multiple.

(a) $2 \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

Answer: $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$

(b) $-3 \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

Answer: $\begin{pmatrix} -6 \\ -12 \end{pmatrix}$

(c) $0.5 \begin{pmatrix} 8 \\ -6 \end{pmatrix}$

Answer: $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

(d) $0.25 \begin{pmatrix} 12 \\ 8 \end{pmatrix}$

Answer: $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

4 $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$. Work out each vector.

(a) $\mathbf{a} + \mathbf{b}$

Answer: $\begin{pmatrix} 1 \\ 7 \end{pmatrix}$

(b) $2a + b$

Answer: $\begin{pmatrix} 3 \\ 10 \end{pmatrix}$

(c) $3a - 2b$

Answer: $\begin{pmatrix} 8 \\ 1 \end{pmatrix}$

(d) $a - 3b$

Answer: $\begin{pmatrix} 5 \\ -9 \end{pmatrix}$

5 In each case, say whether the two vectors are parallel.

(a) $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

[illegible]

(b) $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$

Answer: Parallel ($\mathbf{b} = -2\mathbf{a}$)

(c) $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 8 \\ 3 \end{pmatrix}$

Answer: Not parallel

(d) $\begin{pmatrix} 5 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -10 \\ 0 \end{pmatrix}$

Answer: Parallel ($\mathbf{b} = -2\mathbf{a}$)

6 Work out the magnitude of each vector. Give whole-number answers.

(a) $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

Answer: 5

(b) $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$

Answer: 13

(c) $\begin{pmatrix} 8 \\ 6 \end{pmatrix}$

Answer: 10

(d) $\begin{pmatrix} -7 \\ 24 \end{pmatrix}$

Answer: 25

7 Work out the magnitude of each vector, correct to 1 decimal place.

(a) $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$

Answer: 5.4

(b) $\begin{pmatrix} -3 \\ -7 \end{pmatrix}$

Answer: 7.6

(c) $\begin{pmatrix} 6 \\ -1 \end{pmatrix}$

Answer: 6.1

8 $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$. Work out $|\mathbf{a} + \mathbf{b}|$ correct to 1 decimal place.

Answer: 7.1

9 In 1 hour a plane would fly $\begin{pmatrix} 240 \\ 180 \end{pmatrix}$ km (east, north) in still air. The wind blows it by $\begin{pmatrix} -40 \\ -60 \end{pmatrix}$ km. Work out the plane's actual movement as a vector and the distance it actually travels in 1 hour, correct to 1 decimal place.

Answer: $\begin{pmatrix} 200 \\ 120 \end{pmatrix}$; 233.2 km

10 $\mathbf{a} = \begin{pmatrix} 6 \\ -9 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$. **Show that $\mathbf{a}$ is parallel to $\mathbf{b}$, and say how many times as long $\mathbf{a}$ is.**

Answer: $\mathbf{a} = -3\mathbf{b}$, so parallel; 3 times as long

11 Zara says that $|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$ for all vectors. Use $\mathbf{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ to show she is wrong.

Answer: $|\mathbf{a} + \mathbf{b}| = 5$ but $|\mathbf{a}| + |\mathbf{b}| = 7$

12 The vector $\begin{pmatrix} k \\ 6 \end{pmatrix}$ is parallel to $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Work out the value of k .

Answer: $k = 4$

13 Challenge. $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Find a vector that is parallel to $\mathbf{a}$, in the same direction, with magnitude 10.

Answer: $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$