

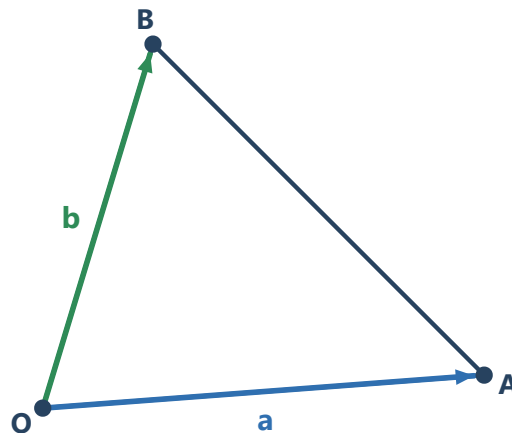
GCSE Maths: Vector geometry and proof (Higher): answers

GCSE Maths - Loci, Bearings and Vectors - Higher - Lesson 22.9

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at websink.site/gcse/maths/ch22-constructions-loci-bearings-vectors/higher/vector-geometry-and-proof/

Exercise 22L Vector routes, midpoints and ratios Non-calculator

- 1 OAB is a triangle. $OA = \mathbf{a}$ and $OB = \mathbf{b}$. Find each vector in terms of $\mathbf{a}$ and $\mathbf{b}$.



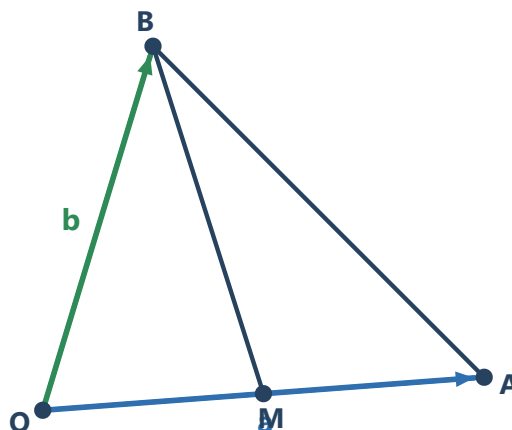
- (a) $\vec{AB}$

Answer: $\mathbf{b} - \mathbf{a}$

- (b) $\vec{BA}$

Answer: $\mathbf{a} - \mathbf{b}$

- 2 OAB is a triangle with $OA = \mathbf{a}$ and $OB = \mathbf{b}$. M is the midpoint of OA. Find in terms of $\mathbf{a}$ and $\mathbf{b}$: (a) OM, (b) MB, (c) MA.



- (a) OM

Answer: $\frac{1}{2}\mathbf{a}$

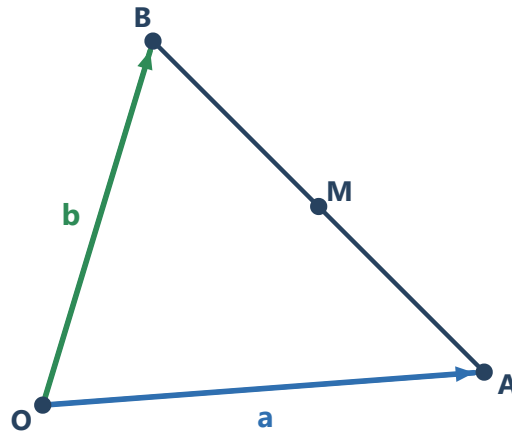
(b) MB

Answer: $\mathbf{b} - \frac{1}{2}\mathbf{a}$

(c) MA

Answer: $\frac{1}{2}\mathbf{a}$

- 3 OAB is a triangle with $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$. M is the midpoint of AB. Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, (a) AM, (b) OM, (c) MB.



(a) AM

Answer: $\frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$

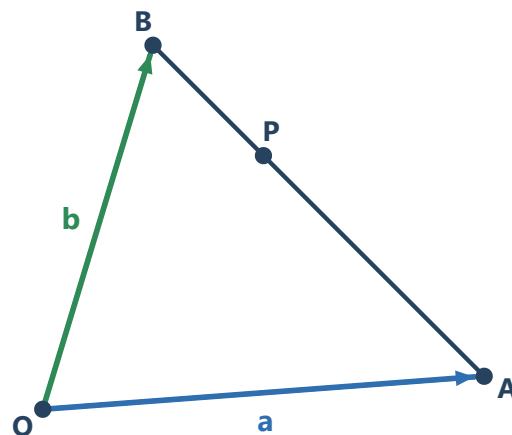
(b) OM

Answer: $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

(c) MB

Answer: $\frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$

- 4 OAB is a triangle with $\mathbf{OA} = \mathbf{a}$ and $\mathbf{OB} = \mathbf{b}$. P is the point on AB such that $\mathbf{AP} : \mathbf{PB} = 2 : 1$. Find OP in terms of $\mathbf{a}$ and $\mathbf{b}$.

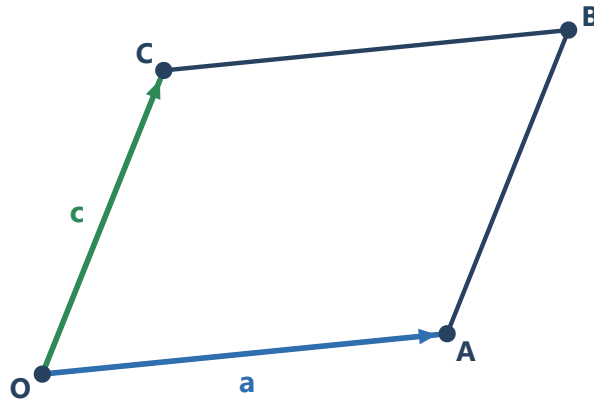


Answer: $\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

- 5 OAB is a triangle with $OA = \mathbf{a}$ and $OB = \mathbf{b}$. Q is the point on AB such that $AQ : QB = 3 : 2$. Find OQ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Answer: $\frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$

- 6 OABC is a parallelogram with $OA = \mathbf{a}$ and $OC = \mathbf{c}$. Find in terms of $\mathbf{a}$ and $\mathbf{c}$: (a) OB, (b) AC, (c) CA.



- (a) OB

Answer: $\mathbf{a} + \mathbf{c}$

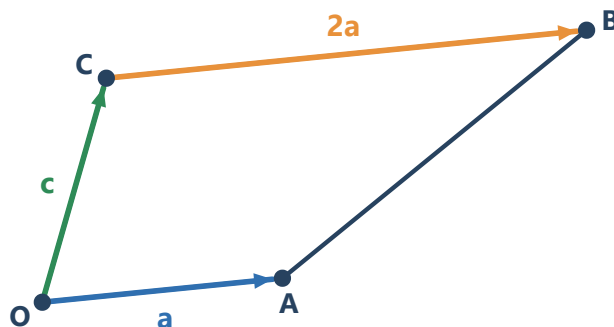
- (b) AC

Answer: $\mathbf{c} - \mathbf{a}$

- (c) CA

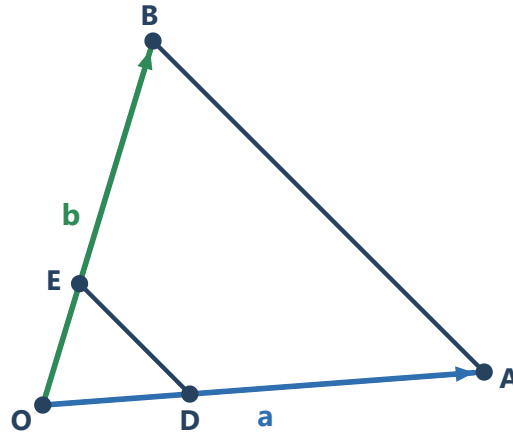
Answer: $\mathbf{a} - \mathbf{c}$

- 7 OABC is a trapezium. $OA = \mathbf{a}$, $OC = \mathbf{c}$ and $CB = 2\mathbf{a}$. Find AB in terms of $\mathbf{a}$ and $\mathbf{c}$.



Answer: $\mathbf{a} + \mathbf{c}$

- 8 OAB is a triangle with $OA = \mathbf{a}$ and $OB = \mathbf{b}$. D is the point on OA with $OD = \frac{1}{3}\mathbf{a}$. E is the point on OB with $OE = \frac{1}{3}\mathbf{b}$. Show that DE is parallel to AB.



Answer: $DE = \frac{1}{3}AB$, so parallel

- 9** O is the origin. $OP = \mathbf{a} + \mathbf{b}$, $OQ = 3\mathbf{a} + 2\mathbf{b}$ and $OR = 5\mathbf{a} + 3\mathbf{b}$. **Show that** P, Q and R are collinear.

Answer: $\vec{QR} = 1\vec{PQ}$, so collinear

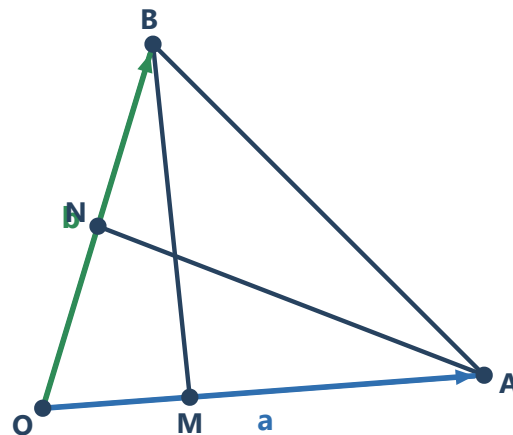
- 10** O is the origin. $OP = 2\mathbf{a} + \mathbf{b}$, $OQ = -\mathbf{a} + 2\mathbf{b}$ and $OR = -4\mathbf{a} + 3\mathbf{b}$. **Show that** P, Q and R are collinear, and write down the ratio PQ : QR.

Answer: $\vec{QR} = 1\vec{PQ}$, so collinear; PQ : QR = 1 : 1

- 11** O is the origin. $OP = \mathbf{a} + 2\mathbf{b}$, $OQ = 2\mathbf{a} + 5\mathbf{b}$ and $OR = 4\mathbf{a} + 9\mathbf{b}$. Show that P, Q and R are **not** collinear.

Answer: Not collinear

- 12** OAB is a triangle with $OA = \mathbf{a}$ and $OB = \mathbf{b}$. M is the point on OA with $OM = \frac{1}{3}\mathbf{a}$. N is the midpoint of OB. AN and BM meet at X. Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.



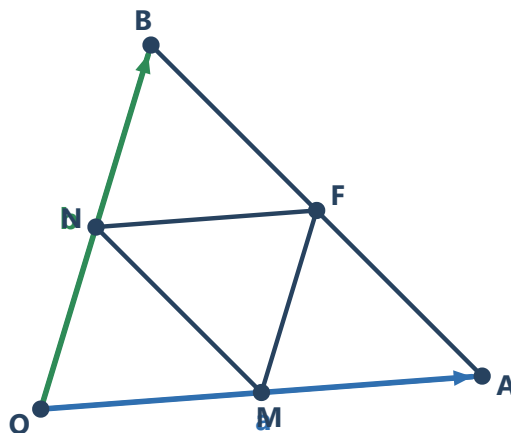
Answer: $\frac{1}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$

- 13** OAB is a triangle with $OA = \mathbf{a}$ and $OB = \mathbf{b}$. M is the midpoint of OA and N is the midpoint of OB. AN and BM meet at X. **Show that** $OX = \frac{1}{3}(\mathbf{a} + \mathbf{b})$, and hence that X is two thirds of the way along the median from A to N.

Answer: $\frac{1}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$; AX : XN = 2 : 1

Exercise 22M Vector proof Non-calculator

- 1 OAB is a triangle with $OA = \mathbf{a}$ and $OB = \mathbf{b}$. M, N and F are the midpoints of OA, OB and AB. Prove that (a) MF is parallel to OB, (b) NF is parallel to OA.



- (a) MF is parallel to OB and half its length.

Answer: $MF = \frac{1}{2}\mathbf{b}$

- (b) NF is parallel to OA and half its length.

Answer: $NF = \frac{1}{2}\mathbf{a}$

- 2 OABC is a quadrilateral. $OA = \mathbf{a}$, $OC = \frac{1}{2}\mathbf{a} + \mathbf{b}$ and $CB = 2\mathbf{a}$. Work out AB, and show that OABC is a trapezium.

Answer: $AB = \frac{3}{2}\mathbf{a} + \mathbf{b}$; OA and CB are parallel ($CB = 2\mathbf{a}$)

- 3 O is the origin. $OP = \mathbf{a} + 2\mathbf{b}$, $OQ = 3\mathbf{a} + 4\mathbf{b}$ and $OR = 5\mathbf{a} + 6\mathbf{b}$. Show that Q is the midpoint of PR.

Answer: $PQ = QR = 2\mathbf{a} + 2\mathbf{b}$

- 4 OAB is a triangle with $OA = 4\mathbf{a}$ and $OB = 4\mathbf{b}$. P is on AB with $AP : PB = 1 : 3$. Find OP in terms of $\mathbf{a}$ and $\mathbf{b}$.

Answer: $3\mathbf{a} + \mathbf{b}$

- 5 **Challenge.** ABCD is any quadrilateral. The position vectors of A, B, C, D (from an origin O) are $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$. P, Q, R and S are the midpoints of AB, BC, CD and DA. **Prove** that PQRS is a parallelogram.

Answer: $PQ = SR = \frac{1}{2}(\mathbf{c} - \mathbf{a})$, so PQRS is a parallelogram

- 6 **Challenge.** In triangle OAB, $OA = \mathbf{a}$ and $OB = \mathbf{b}$. The point X has $OX = \frac{1}{3}(\mathbf{a} + \mathbf{b})$. M is the midpoint of OA, and the point M, X and B are collinear. Show that $BX : XM = 2 : 1$.

Answer: $BX = 2 XM$