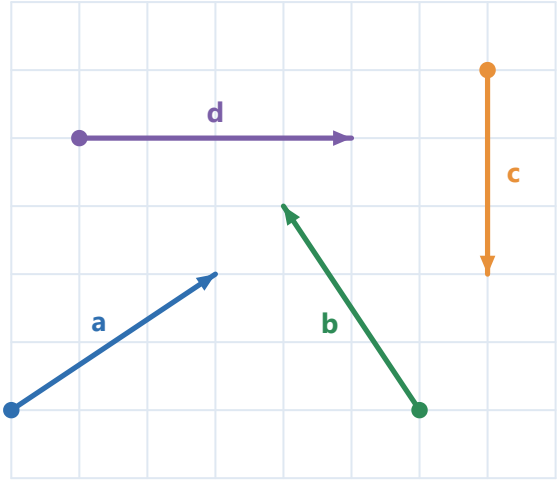
GCSE Maths: Vectors: column vectors, addition and scalar multiples (Higher): answers

GCSE Maths - Loci, Bearings and Vectors - Higher - Lesson 22.8

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at **websink.site/gcse/maths/ch22-constructions-loci-bearings-vectors/higher/vectors-column-vectors-addition-and-scalar-multiples/**

Exercise 22J Column vectors and translations Non-calculator

1 Write down each vector as a column vector.



(a) **a**

Answer: $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

(b) **b**

Answer: $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

(c) **c**

Answer: $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$

(d) **d**

Answer: $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

2 Write each movement as a column vector.

(a) 3 right and 2 up

Answer: $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

(b) 4 left and 1 up

Answer: $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$

(c) 2 right and 5 down

Answer: $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$

(d) 6 left

Answer: $\begin{pmatrix} -6 \\ 0 \end{pmatrix}$

3 **Work out the column vector from the first point to the second point.**

(a) A(1, 2) to B(4, 6)

Answer: $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(b) A(-2, 3) to B(2, -1)

Answer: $\begin{pmatrix} 4 \\ -4 \end{pmatrix}$

(c) A(5, 4) to B(1, 4)

Answer: $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$

(d) A(0, -3) to B(-4, -6)

Answer: $\begin{pmatrix} -4 \\ -3 \end{pmatrix}$

4 **Work out the coordinates of the end point B.**

(a) A(2, 1), vector from A to B is $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

Answer: (5, 5)

(b) A(-1, 5), vector from A to B is $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$

Answer: (3, 3)

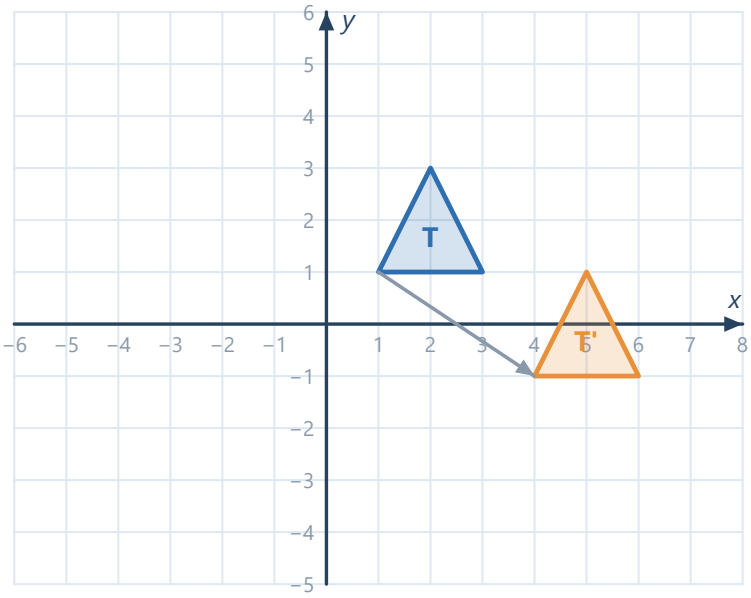
(c) A(0, 0), vector from A to B is $\begin{pmatrix} -3 \\ -6 \end{pmatrix}$

Answer: $(-3, -6)$

(d) A(6, -2), vector from A to B is $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$

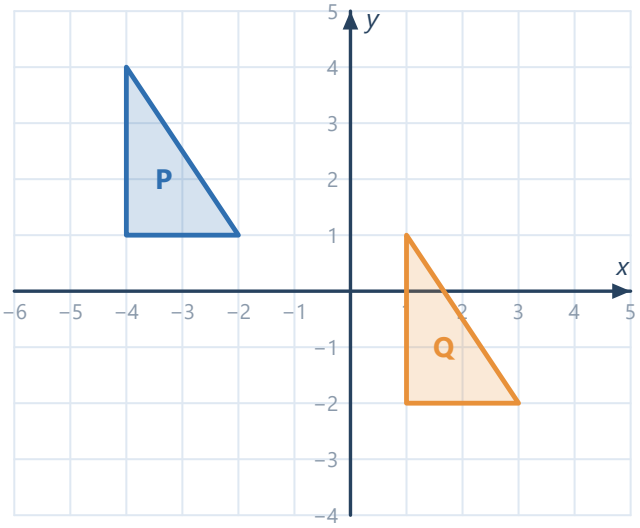
Answer: (4, 3)

5 **Triangle T has vertices A(1, 1), B(3, 1) and C(2, 3). It is translated by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ to give T′. Write the coordinates of A′, B′ and C′.**



Answer: A′ (4, −1), B′ (6, −1), C′ (5, 1)

6 Shape P is translated to shape Q. Describe the translation using a column vector.



Answer: $\begin{pmatrix} 5 \\ -3 \end{pmatrix}$

7 $\mathbf{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$. Write down each vector.

(a) $-\mathbf{a}$

Answer: $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$

(b) $2\mathbf{a}$

Answer: $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$

(c) $-3\mathbf{a}$

Answer: $\begin{pmatrix} -9 \\ 6 \end{pmatrix}$

8 A drone moves $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$, then $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$, then $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$ (in metres). Work out its total movement as a column vector and its distance from where it started.

Answer: $\begin{pmatrix} 7 \\ 0 \end{pmatrix}$, 7 m

9 ABCD is a parallelogram with A at (1, 1). The vector from A to B is $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and the vector from A to D is $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$. Write down (a) the vector from B to C, (b) the vector from D to C, (c) the coordinates of C.

Answer: (a) $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$, (b) $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$, (c) (6, 5)

10 Are the vectors $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ equal? Give a reason.

Answer: No. They have different numbers, so different directions.

11 Triangle T is translated by $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$, and then the image is translated by $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$. **Show that** the final triangle is in the same position as T.

Answer: $\begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

12 A translation maps the point A(2, 3) to A′(7, 1). B is the point (5, −4). Work out the coordinates of B′, the image of B under the same translation.

Answer: (10, −6)

13 A ship sails 6 km east and then 8 km north. Write its movement as a column vector and find how far it is from its start.

Answer: $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$, 10 km

14 **Challenge.** A translation moves the point P(1, 2) to Q and then Q is moved by a second translation $\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ to R(4, 1). Work out the coordinates of Q and the vector from P to Q.

Answer: Q(7, −4); $\begin{pmatrix} 6 \\ -6 \end{pmatrix}$

Exercise 22K Combining vectors: addition, scalar multiples and magnitude

Calculator allowed

1 Work out each sum.

(a) $\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

Answer: $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$

(b) $\begin{pmatrix} -1 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

Answer: $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(c) $\begin{pmatrix} 6 \\ -4 \end{pmatrix} + \begin{pmatrix} -2 \\ -2 \end{pmatrix}$

Answer: $\begin{pmatrix} 4 \\ -6 \end{pmatrix}$

(d) $\begin{pmatrix} 0 \\ 7 \end{pmatrix} + \begin{pmatrix} -5 \\ 3 \end{pmatrix}$

Answer: $\begin{pmatrix} -5 \\ 10 \end{pmatrix}$

2 Work out each difference.

(a) $\begin{pmatrix} 5 \\ 7 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Answer: $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(b) $\begin{pmatrix} 1 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

Answer: $\begin{pmatrix} -3 \\ -7 \end{pmatrix}$

(c) $\begin{pmatrix} -3 \\ 6 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

Answer: $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$

(d) $\begin{pmatrix} 2 \\ -4 \end{pmatrix} - \begin{pmatrix} -6 \\ -9 \end{pmatrix}$

Answer: $\begin{pmatrix} 8 \\ 5 \end{pmatrix}$

3 Work out each scalar multiple.

(a) $2\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

Answer: $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$

(b) $-3\begin{pmatrix} 2 \\ 4 \end{pmatrix}$

Answer: $\begin{pmatrix} -6 \\ -12 \end{pmatrix}$

(c) $0.5\begin{pmatrix} 8 \\ -6 \end{pmatrix}$

Answer: $\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

(d) $0.25\begin{pmatrix}12\\8\end{pmatrix}$

Answer: $\begin{pmatrix}3\\2\end{pmatrix}$

4 $\mathbf{a} = \begin{pmatrix}2\\3\end{pmatrix}$, $\mathbf{b} = \begin{pmatrix}-1\\4\end{pmatrix}$. Work out each vector.

(a) $\mathbf{a} + \mathbf{b}$

Answer: $\begin{pmatrix}1\\7\end{pmatrix}$

(b) $2\mathbf{a} + \mathbf{b}$

Answer: $\begin{pmatrix}3\\10\end{pmatrix}$

(c) $3\mathbf{a} - 2\mathbf{b}$

Answer: $\begin{pmatrix}8\\1\end{pmatrix}$

(d) $\mathbf{a} - 3\mathbf{b}$

Answer: $\begin{pmatrix}5\\-9\end{pmatrix}$

5 In each case, say whether the two vectors are parallel.

(a) $\begin{pmatrix}3\\6\end{pmatrix}$ and $\begin{pmatrix}1\\2\end{pmatrix}$

Answer: Parallel ($\mathbf{b} = 0.3333333333333333333333333333333333333333333333333333333333333333\mathbf{a}$)

(b) $\begin{pmatrix}2\\-3\end{pmatrix}$ and $\begin{pmatrix}-4\\6\end{pmatrix}$

Answer: Parallel ($\mathbf{b} = -2\mathbf{a}$)

(c) $\begin{pmatrix}4\\1\end{pmatrix}$ and $\begin{pmatrix}8\\3\end{pmatrix}$

Answer: Not parallel

(d) $\begin{pmatrix}5\\0\end{pmatrix}$ and $\begin{pmatrix}-10\\0\end{pmatrix}$

Answer: Parallel ($\mathbf{b} = -2\mathbf{a}$)

6 Work out the magnitude of each vector. Give whole-number answers.

(a) $\begin{pmatrix}3\\4\end{pmatrix}$

Answer: 5

(b) $\begin{pmatrix}5\\12\end{pmatrix}$

Answer: 13

(c) $\begin{pmatrix} 8 \\ 6 \end{pmatrix}$

Answer: 10

(d) $\begin{pmatrix} -7 \\ 24 \end{pmatrix}$

Answer: 25

7 Work out the magnitude of each vector, correct to 1 decimal place.

(a) $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$

Answer: 5.4

(b) $\begin{pmatrix} -3 \\ -7 \end{pmatrix}$

Answer: 7.6

(c) $\begin{pmatrix} 6 \\ -1 \end{pmatrix}$

Answer: 6.1

8 Work out the magnitude of each vector. Give your answer as a simplified surd.

(a) $\begin{pmatrix} 4 \\ 6 \end{pmatrix}$

Answer: $2\sqrt{13}$

(b) $\begin{pmatrix} 2 \\ 7 \end{pmatrix}$

Answer: $\sqrt{53}$

(c) $\begin{pmatrix} -5 \\ 5 \end{pmatrix}$

Answer: $5\sqrt{2}$

(d) $\begin{pmatrix} 1 \\ -8 \end{pmatrix}$

Answer: $\sqrt{65}$

9 $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$. Work out $|\mathbf{a} + \mathbf{b}|$ correct to 1 decimal place.

Answer: 7.1

10 In 1 hour a plane would fly $\begin{pmatrix} 240 \\ 180 \end{pmatrix}$ km (east, north) in still air. The wind blows it by $\begin{pmatrix} -40 \\ -60 \end{pmatrix}$ km. Work out the plane's actual movement as a vector and the distance it actually travels in 1 hour, correct to 1 decimal place.

Answer: $\begin{pmatrix} 200 \\ 120 \end{pmatrix}$; 233.2 km

11 $\mathbf{a} = \begin{pmatrix} 6 \\ -9 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$. **Show that a** is parallel to **b**, and say how many times as long **a** is.

Answer: $a = -3b$, so parallel; 3 times as long

12 Zara says that $|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$ for all vectors. Use $\mathbf{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$ to show she is wrong.

> **Answer:** $|a + b| = 5$ but $|a| + |b| = 7$

13 The vector $\begin{pmatrix} k \\ 6 \end{pmatrix}$ is parallel to $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$. Work out the value of k.

> **Answer:** $k = 4$

14 $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$. Find the numbers m and n such that $m\mathbf{a} + n\mathbf{b} = \begin{pmatrix} 11 \\ 1 \end{pmatrix}$.

> **Answer:** $m = 2$, $n = 3$

15 **Challenge.** $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Find a vector that is parallel to $\mathbf{a}$, in the same direction, with magnitude 10.

> **Answer:** $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$

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