

GCSE Maths: Conditional probability and probability with algebra (Higher): answers

GCSE Maths - Probability - Higher - Lesson 23.9

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at websink.site/gcse/maths/ch23-probability/higher/conditional-probability-and-probability-with-algebra/

Exercise 23M Conditional probability Non-calculator

- 1 The table shows how 35 students travel to school. Find the probability that...

	Walk	Bus	Total
Boys	12	6	18
Girls	9	8	17
Total	21	14	35

- (a) the student is a boy, given that the student takes the bus

Answer: $\frac{3}{7}$

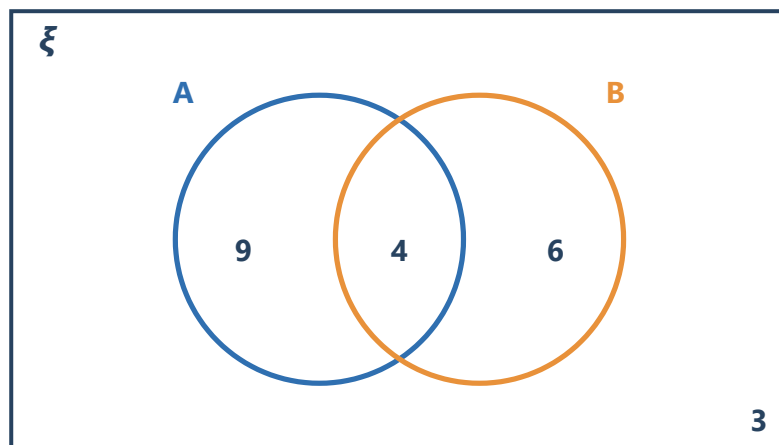
- (b) the student walks to school, given that the student is a girl

Answer: $\frac{9}{17}$

- (c) the student takes the bus, given that the student is a boy

Answer: $\frac{1}{3}$

- 2 The Venn diagram shows 22 students: A studies Spanish and B studies French. A student is chosen at random. Find...



- (a) $P(A | B)$

Answer: $\frac{2}{5}$

(b) $P(B | A)$

Answer: $\frac{4}{13}$

(c) $P(A | B')$

Answer: $\frac{3}{4}$

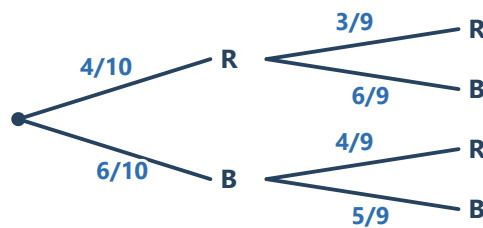
3 A fair die is rolled. Given that the score is even, find the probability that the score is greater than 3.

Answer: $\frac{2}{3}$

4 A card is chosen from a standard pack of 52 cards. Given that the card is a heart, find the probability that it is a king.

Answer: $\frac{1}{13}$

5 A bag has 4 red and 6 blue counters. Two counters are taken at random without replacement. Find...



(a) the probability that the second counter is red, given that the first was red

Answer: $\frac{1}{3}$

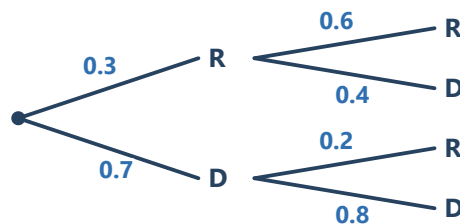
(b) the probability that the first counter was red, given that the second is red

Answer: $\frac{1}{3}$

(c) the probability that the counters are different colours, given that the second is blue

Answer: $\frac{4}{9}$

6 The probability of rain on Monday is 0.3. If it rains on Monday, the probability of rain on Tuesday is 0.6. If it is dry on Monday, the probability of rain on Tuesday is 0.2. Find the probability that...



(a) it rains on both days

Answer: 0.18

- (b) it rains on Tuesday

Answer: 0.32

- (c) it rained on Monday, given that it rains on Tuesday

Answer: $\frac{9}{16}$

- 7 A test is given to 2000 people. 1% have a disease. The test is positive for 95% of people with the disease and for 4% of people without it. Use a frequency tree to find the probability that a person with a positive test has the disease. Give your answer to 2 decimal places.

Answer: 0.19

- 8 Zara says: " $P(A | B)$ is always the same as $P(B | A)$." In a Venn diagram $n(A) = 13$, $n(B) = 10$ and $n(A \cap B) = 4$. Show that Zara is wrong.

Answer: $P(A | B) = 2/5$ and $P(B | A) = 4/13$

- 9 $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \text{ and } B) = 0.1$. Show that A and B are not independent.

Answer: $P(A | B) = 0.25$, not 0.5

- 10 The table shows 80 students. Find the two probabilities below, and say what they tell you about playing an instrument and being in Year 10.

	Instrument	None	Total
Year 10	16	14	30
Year 11	24	26	50
Total	40	40	80

- (a) $P(\text{Year 10} | \text{plays an instrument})$

Answer: $\frac{2}{5}$

- (b) $P(\text{Year 10})$

Answer: $\frac{3}{8}$

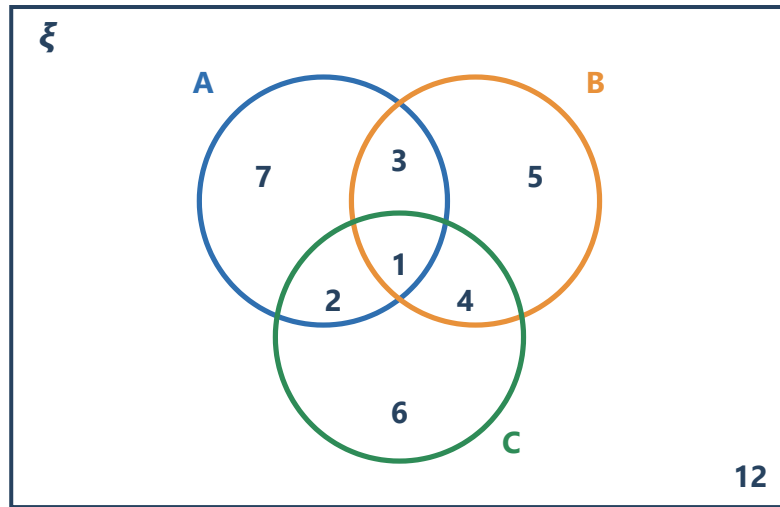
- (c) What the answers tell you

Answer: $2/5$ is greater than $3/8$, so the events are not independent

- 11 A bag contains 3 red and 5 blue counters. A counter is picked at random and put aside. A second counter is then picked. Given that the second counter is blue, find the probability that the first counter was red.

Answer: $\frac{3}{7}$

- 12 The Venn diagram shows 40 students who travel by bus (A), bike (B) or car (C). Find...



(a) $P(A | B)$

Answer: $\frac{4}{13}$

(b) $P(C | A')$

Answer: $\frac{10}{27}$

(c) $P(A \cap B | C)$

Answer: $\frac{1}{13}$

Exercise 23N Probability and algebra Non-calculator

- 1 A bag has 3 red counters and x blue counters. $P(\text{blue}) = \frac{5}{8}$. Form an equation and find the value of x .

Answer: 5

- 2 A bag has x red counters, $2x$ blue counters and 6 green counters. $P(\text{green}) = \frac{1}{4}$. Find the value of x and the number of counters in the bag.

Answer: $x = 6$ and 24 counters

- 3 The probabilities that a spinner lands on A, B, C and D are 0.3, x , $2x$ and $x + 0.1$. Find the value of x and the probability that it lands on D.

Answer: $x = 0.15$ and $P(D) = 0.25$

- 4 A bag has n counters. 4 of them are red and the rest are blue. Two counters are picked at random without replacement. The probability that both are red is $\frac{1}{11}$. Show that $n^2 - n - 132 = 0$ and find the value of n .

Answer: $n = 12$

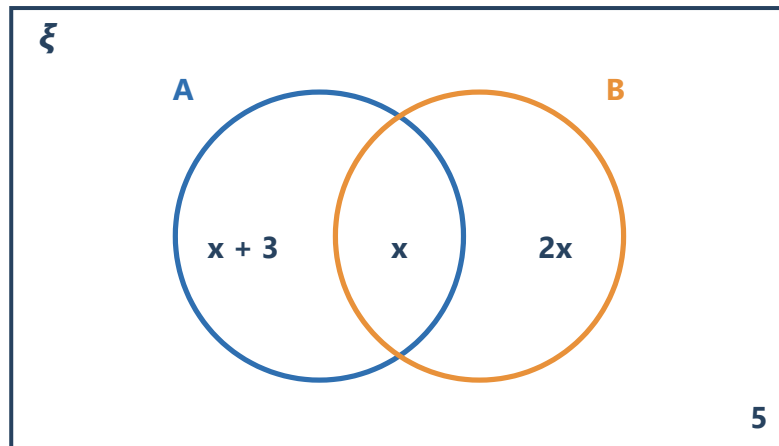
- 5 A bag has x red counters and 6 blue counters. A counter is picked, replaced, and a second counter is picked. The probability that both are red is $\frac{1}{4}$. Find the value of x .

Answer: 6

- 6 A bag has 3 red counters and x blue counters. Two counters are picked without replacement. $P(\text{both red}) = \frac{1}{5}$. Show that $x^2 + 5x - 24 = 0$ and find x .

Answer: $x = 3$

- 7 The Venn diagram shows 40 students. Find the value of x and then find the probability that a student chosen at random is in both A and B.



Answer: $x = 8$ and the probability is $1/5$

- 8 $P(A) = x$, $P(B) = 2x$, $P(A \cap B) = 0.1$ and $P(A \cup B) = 0.5$. Find the value of x .

Answer: 0.2

- 9 The probability that Kim is late on any day is p , independently each day. The probability that she is late on both of two days is 0.09. Find p , and then the probability that she is late on exactly one of the two days.

Answer: $p = 0.3$ and 0.42

- 10 A bag contains red and blue counters. $P(\text{red}) = \frac{3}{5}$. Four more blue counters are added and then $P(\text{red}) = \frac{1}{2}$. Find the original number of counters of each colour.

Answer: 12 red and 8 blue

- 11 $P(A \text{ and } B) = 0.12$ and $P(B) = x$. Given that $P(A | B) = 0.4$, find the value of x .

Answer: 0.3

- 12 The probability that Mia wins her first game is p . If she wins, the probability she wins her second game is 0.7. If she loses, the probability she wins her second game is 0.4. The probability that she wins the second game is 0.55. Find p .

Answer: 0.5

- 13 A bag has x red and y blue counters. Two counters are picked without replacement. Prove that the probability that they are different colours is $\frac{2xy}{(x+y)(x+y-1)}$.

Answer: Proved: two routes, each with probability xy over $(x+y)(x+y-1)$