

GCSE Maths: Gradients of curves and rates of change (Higher): answers

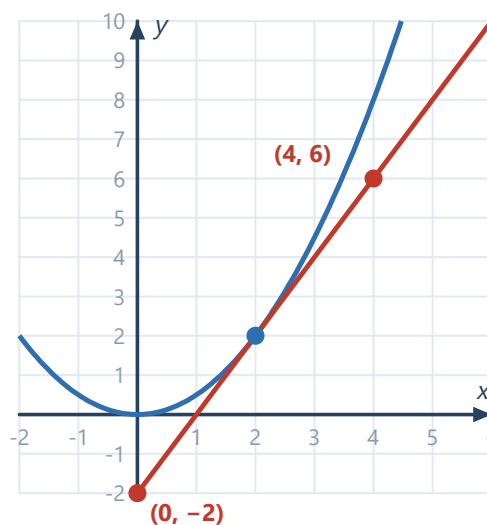
GCSE Maths - Functions and Advanced Graphs - Higher - Lesson 25.8

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at websink.site/gcse/maths/ch25-functions-advanced-graphs/higher/gradients-of-curves-and-rates-of-change/

Exercise 25H Gradients of curves and rates of change Non-calculator

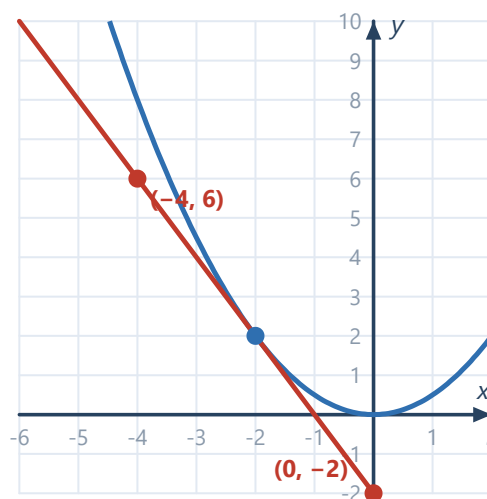
- 1 In each diagram a tangent has been drawn to the curve at the marked point. Use the two points shown on the tangent to estimate the gradient of the curve at that point.

(a) $y = x^2 \div 2$ at $x = 2$



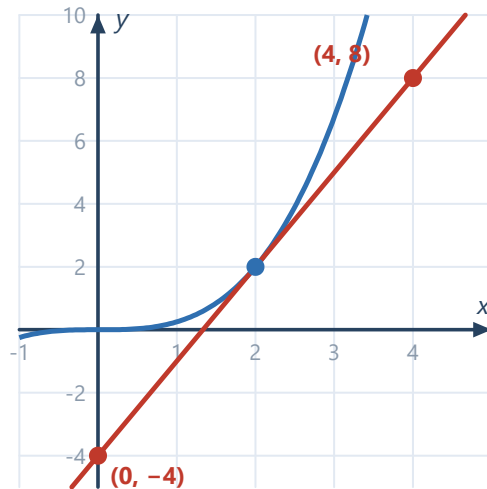
Answer: 2

(b) $y = x^2 \div 2$ at $x = -2$



Answer: -2

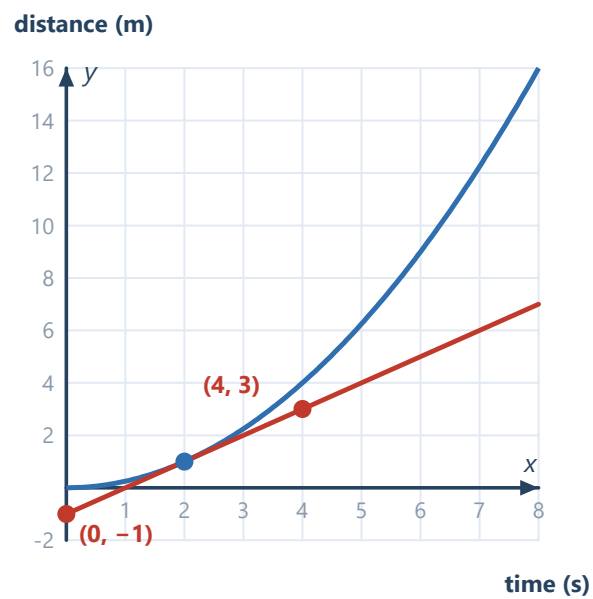
(c) $y = x^3 \div 4$ at $x = 2$



Answer: 3

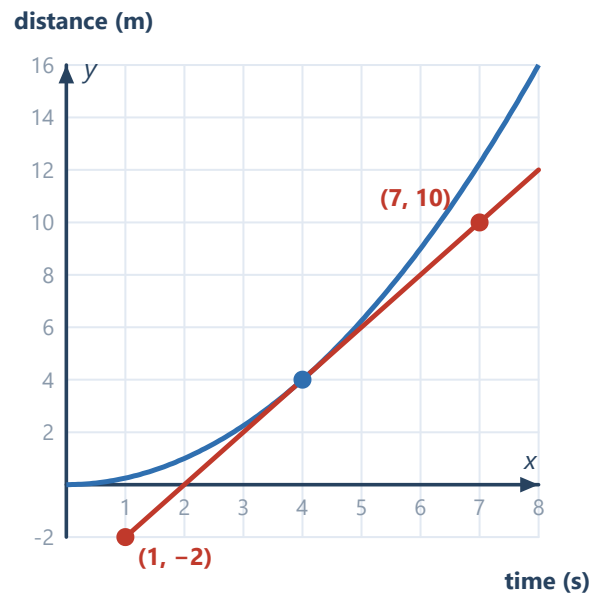
- 2 The distance, s metres, travelled by a cyclist in t seconds is given by $s = t^2 \div 4$. A tangent has been drawn on the graph. Use it to estimate the cyclist's speed after

(a) 2 seconds



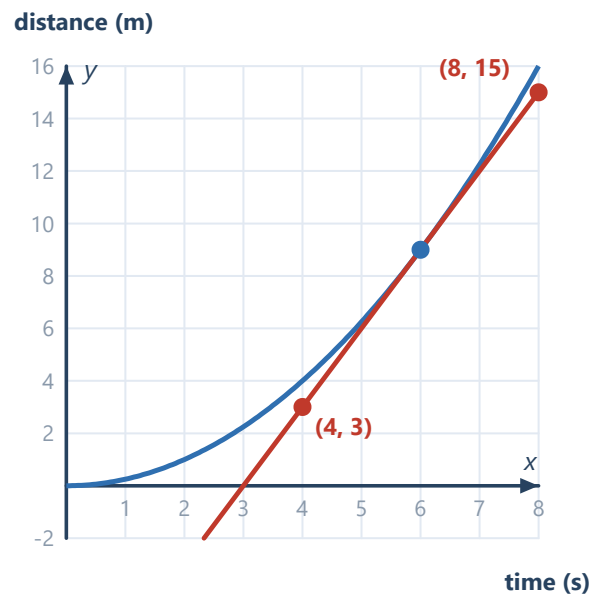
Answer: 1

(b) 4 seconds



Answer: 2

(c) 6 seconds



Answer: 3

3 A rocket's height is $s = t^2$ metres after t seconds. Work out its average speed between

(a) $t = 0$ and $t = 2$

Answer: 2 m/s

(b) $t = 2$ and $t = 4$

Answer: 6 m/s

(c) $t = 4$ and $t = 8$

Answer: 12 m/s

4 The value of an investment is $V = 100 \times 1.1^t$ pounds after t years. Work out the average rate of change of its value between

- (a) $t = 0$ and $t = 2$

Answer: 10.5 pounds/year

- (b) $t = 2$ and $t = 4$

Answer: 12.705 pounds/year

- 5 A cyclist's distance is $s = 0.5t^2 + 2t$ metres after t seconds. Estimate her speed at $t = 4$ by working out the average speeds over the intervals from $t = 4$ to 5, 4.5, 4.1 and 4.01.

Answer: 6

- 6 For each curve, find the gradient of the chord from $x = a$ to $x = a + h$, simplified, and use it to write down the gradient of the curve at $x = a$.

- (a) $y = x^2$, $a = 3$

Answer: $h + 6$

- (b) $y = x^3$, $a = 1$

Answer: $h^2 + 3h + 3$

- (c) $y = x^2$, $a = 5$

Answer: $h + 10$

- (d) $y = x^3$, $a = -2$

Answer: $h^2 - 6h + 12$

- 7 A car's distance is $s = t^2$ metres after t seconds. (a) Work out its average speed over the first 4 seconds. (b) Work out the gradient of the chord from $t = 4$ to $t = 4.01$ and use it to estimate the speed at $t = 4$. (c) Explain why the two answers differ.

Answer: (a) 4 m/s (b) about 8 m/s (c) the car speeds up

- 8 The population of a colony of bacteria is $P = 1000 \times 1.1^t$ after t hours. (a) Work out the average rate of increase between $t = 2$ and $t = 5$. (b) Estimate the rate of increase at $t = 3$ using the interval from $t = 3$ to $t = 3.01$. Give answers to the nearest whole number of bacteria per hour.

Answer: (a) about 134 per hour (b) about 127 per hour

- 9 For each graph, give the units of the gradient and say what the gradient represents.

- (a) A graph of cost (£) against number of items bought has gradient 3.5.

Answer: each extra item costs £3.50 (price per item)

- (b) A graph of speed (m/s) against time (s) has gradient 2.

Answer: the acceleration is 2 m/s²

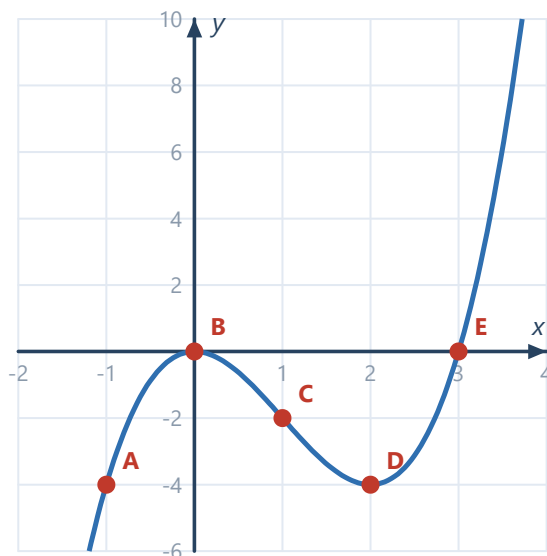
- (c) A graph of distance (km) against time (hours) has gradient 60.

Answer: the speed is 60 km/h

- (d) A graph of litres of fuel left against distance (km) has gradient -0.08 .

Answer: the car uses 0.08 litres of fuel per km (the minus sign means the fuel is decreasing)

- 10 The curve is $y = x^2(x - 3)$. Five points are marked. Write down the points where the gradient of the curve is



- (a) zero

Answer: B and D

- (b) positive

Answer: A and E

- (c) negative

Answer: C

- 11 A student says: "The average speed of the train over the whole journey is 80 km/h, so the speed at the half-way point must be 80 km/h." Explain why the student may be wrong.

Answer: An average over an interval does not tell you the instantaneous speed

- 12 The curve $y = x^2 - 4x$ has its lowest point at $(2, -4)$. Explain why the gradient of the curve is 0 at that point.

Answer: The tangent at the lowest point is horizontal

- 13 For $y = x^2$, compare the gradient of the chord from $x = 3$ to $x = 5$ with the gradient of the tangent at $x = 3$ (which is 6). Is the chord gradient bigger or smaller, and why does that make sense on the graph?

Answer: Chord gradient 8 is bigger than 6

- 14 Use the chord gradient $2a + h$ for the curve $y = x^2$ between $x = a$ and $x = a + h$ to find the value of x at which the gradient of the curve is 10.

Answer: $x = 5$