

GCSE Maths: Graph transformations (Higher): answers

GCSE Maths - Functions and Advanced Graphs - Higher - Lesson 25.4

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at websink.site/gcse/maths/ch25-functions-advanced-graphs/higher/graph-transformations/

Exercise 25D Graph transformations Non-calculator

- 1 The graph of $y = f(x)$ passes through A(1, 0), B(2, -1), C(3, 0) and D(0, 3). Find the coordinates of the images of A, B, C and D on the graph of

(a) $y = f(x) + 3$

Answer: A'(1, 3), B'(2, 2), C'(3, 3), D'(0, 6)

(b) $y = f(x - 2)$

Answer: A'(3, 0), B'(4, -1), C'(5, 0), D'(2, 3)

(c) $y = -f(x)$

Answer: A'(1, 0), B'(2, 1), C'(3, 0), D'(0, -3)

(d) $y = f(-x)$

Answer: A'(-1, 0), B'(-2, -1), C'(-3, 0), D'(0, 3)

- 2 The graph of $y = f(x) = x^3 - 3x^2$ passes through A(0, 0), B(2, -4) and C(3, 0). Find the images of A, B and C on

(a) $y = f(x) - 2$

Answer: A'(0, -2), B'(2, -6), C'(3, -2)

(b) $y = f(x + 1)$

Answer: A'(-1, 0), B'(1, -4), C'(2, 0)

(c) $y = 2f(x)$ (a stretch)

Answer: A'(0, 0), B'(2, -8), C'(3, 0)

(d) $y = f(-x)$

Answer: A'(0, 0), B'(-2, -4), C'(-3, 0)

- 3 $f(x) = x^2 - 4x$. Find, in expanded form, the equation of the graph

(a) $y = f(x) + 3$

Answer: $x^2 - 4x + 3$

(b) $y = f(x - 2)$

Answer: $x^2 - 8x + 12$

(c) $y = -f(x)$

Answer: $-x^2 + 4x$

(d) $y = f(-x)$

Answer: $x^2 + 4x$

(e) $y = 2f(x)$

Answer: $2x^2 - 8x$

- 4 The graph of $y = f(x)$ has a minimum point at $(2, -1)$. Write down the coordinates and type of the turning point of

(a) $y = f(x) + 4$

Answer: minimum at $(2, 3)$

(b) $y = f(x - 3)$

Answer: minimum at $(5, -1)$

(c) $y = -f(x)$

Answer: maximum at $(2, 1)$

(d) $y = f(2x)$ (a stretch)

Answer: minimum at $(1, -1)$

- 5 The graph of $y = f(x)$ crosses the x -axis at $x = 1$ and $x = 3$. Write down where the graph of each of these crosses the x -axis.

(a) $y = f(x - 2)$

Answer: $x = 3$ and $x = 5$

(b) $y = f(-x)$

Answer: $x = -3$ and $x = -1$

(c) $y = -f(x)$

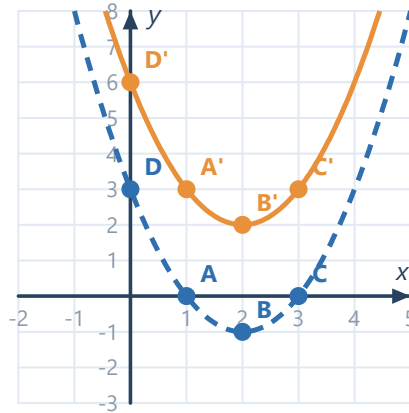
Answer: $x = 1$ and $x = 3$

(d) $y = f(2x)$

Answer: $x = 0.5$ and $x = 1.5$

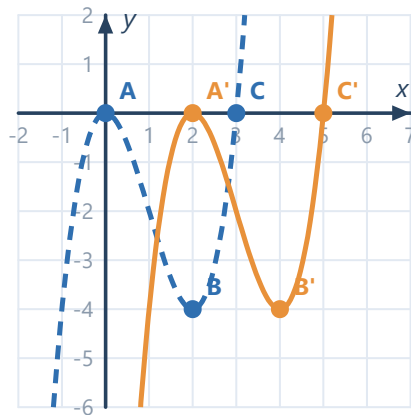
- 6 In each diagram the dashed graph is $y = f(x)$ and the orange graph is a transformation of it. Describe the transformation fully and write the equation of the orange graph in terms of f .

(a)



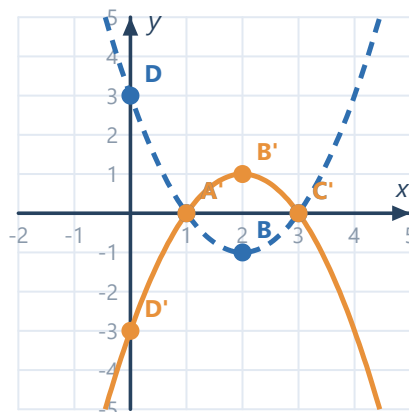
Answer: a translation by the vector $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ (3 units up); $y = f(x) + 3$

(b)



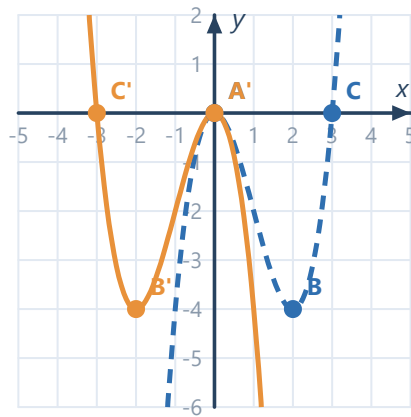
Answer: a translation by the vector $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ (2 units to the right); $y = f(x - 2)$

(c)



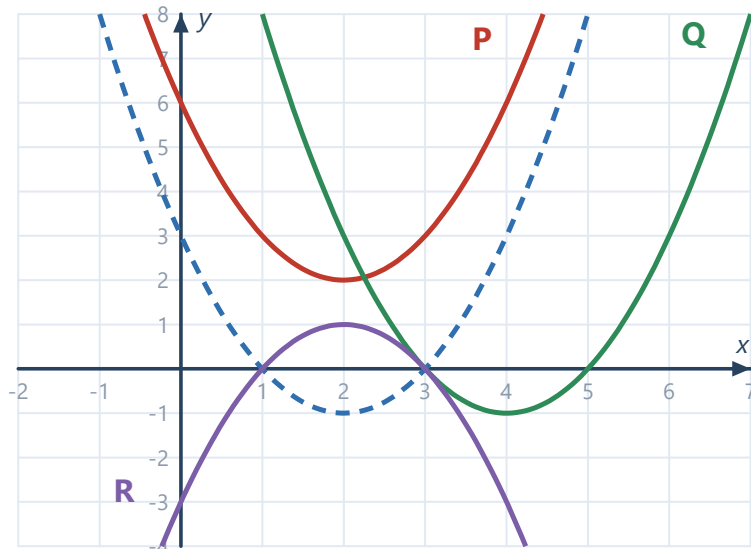
Answer: a reflection in the x-axis; $y = -f(x)$

(d)



Answer: a reflection in the y -axis; $y = f(-x)$

- 7 The dashed graph is $y = f(x) = x^2 - 4x + 3$. The graphs P, Q and R are transformations of it. Match each equation to a graph.



- (a) $y = f(x) + 3$

Answer: P

- (b) $y = f(x - 2)$

Answer: Q

- (c) $y = -f(x)$

Answer: R

- 8 The point (4, 7) is on the graph of $y = f(x)$. Write down the coordinates of the corresponding point on the graph of

- (a) $y = f(x) + 1$

Answer: (4, 8)

(b) $y = f(x - 2)$

Answer: (6, 7)

(c) $y = 2f(x)$

Answer: (4, 14)

(d) $y = f(\frac{1}{2}x)$

Answer: (8, 7)

9 The graph of $y = 1/x$ has asymptotes $x = 0$ and $y = 0$. Write down the equations of the asymptotes of

(a) $y = 1/x + 2$

Answer: $x = 0$ and $y = 2$

(b) $y = 1/(x - 3)$

Answer: $x = 3$ and $y = 0$

(c) $y = 1/(x + 1) - 4$

Answer: $x = -1$ and $y = -4$

(d) $y = 1/(x - 2) + 1$

Answer: $x = 2$ and $y = 1$

10 Sketch the graph of $y = 1/(x - 2) + 1$. Mark the asymptotes and the points where the graph crosses the axes.

Answer: Asymptotes $x = 2, y = 1$; crosses the axes at (1, 0) and (0, 0.5)

11 Complete the square for each quadratic, and write down the coordinates of its turning point.

(a) $y = x^2 + 6x + 11$

Answer: (-3, 2)

(b) $y = x^2 - 4x + 9$

Answer: (2, 5)

(c) $y = x^2 + 8x + 20$

Answer: (-4, 4)

(d) $y = x^2 - 2x - 3$

Answer: (1, -4)

12 Describe the transformation that maps the graph of $y = x^2$ onto the graph of $y = (x - 3)^2 + 2$.

Answer: A translation by the vector (3, 2)

13 Explain why the graph of $y = f(x + 3)$ is a translation of 3 units to the **left**, using $f(x) = x^2$ as an example.

Answer: Its root is at $x = -3$, three to the left of the root of x^2 at $x = 0$

14 For $f(x) = x^3 - 3x^2$, show that the graphs of $y = -f(x)$ and $y = f(-x)$ are **not** the same.

Answer: $-f(x) = -x^3 + 3x^2$ but $f(-x) = -x^3 - 3x^2$

- 15** The height, in metres, of a ball x seconds after it is thrown is $h(x) = 8x - 5x^2$. A second ball is thrown in exactly the same way but 1 second later. Find an expression for its height x seconds after the first ball was thrown.

Answer: $h(x - 1) = -5x^2 + 18x - 13$

- 16** The graph of $y = f(x)$ has a minimum at $(3, -2)$. The graph of $y = f(x + a) + b$ has its minimum at $(1, 5)$. Find the values of a and b .

Answer: $a = 2, b = 7$

- 17** The graph of $y = x^2 - 4x + 3$ is reflected in the y -axis and then translated 2 units up. Find the equation of the final graph.

Answer: $y = x^2 + 4x + 5$