

GCSE Maths: Inverse functions (Higher): answers

GCSE Maths - Functions and Advanced Graphs - Higher - Lesson 25.2

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at websink.site/gcse/maths/ch25-functions-advanced-graphs/higher/inverse-functions/

Exercise 25B Inverse functions Non-calculator

1 Find the inverse function $f^{-1}(x)$ of each function.

(a) $f(x) = x + 7$

Answer: $x - 7$

(b) $f(x) = 5x$

Answer: $\frac{x}{5}$

(c) $f(x) = \frac{x}{3}$

Answer: $3x$

(d) $f(x) = -x + 10$

Answer: $-x + 10$

2 Find the inverse function $f^{-1}(x)$ of each function.

(a) $f(x) = 2x + 3$

Answer: $\frac{x - 3}{2}$

(b) $f(x) = 3x - 4$

Answer: $\frac{x + 4}{3}$

(c) $f(x) = \frac{x - 2}{5}$

Answer: $5x + 2$

(d) $f(x) = 4(x + 1)$

Answer: $\frac{x - 4}{4}$

3 This function machine shows f . (a) Draw the inverse machine. (b) Write down $f^{-1}(x)$.



Answer: Add 1, then divide by 4; $f^{-1}(x) = (x + 1) \div 4$

4 $f(x) = 4x - 1$. Find

(a) $f^{-1}(7)$

Answer: 2

(b) $f^{-1}(-5)$

Answer: -1

(c) $f^{-1}(0)$

Answer: 0.25

5 Find the inverse function $f^{-1}(x)$, where $x \neq 0$ or the denominator is not zero.

(a) $f(x) = \frac{8}{x}$

Answer: $\frac{8}{x}$

(b) $f(x) = \frac{6}{x} - 2$

Answer: $\frac{6}{x+2}$

(c) $f(x) = \frac{4}{x+1}$

Answer: $\frac{-x+4}{x}$

(d) $f(x) = \frac{3}{2x}$

Answer: $\frac{3}{2x}$

6 Each function below is used only for the inputs shown, so it has an inverse. Find $f^{-1}(x)$.

(a) $f(x) = x^2 + 3, \quad x \geq 0$

Answer: $\sqrt{x-3}$

(b) $f(x) = (x-1)^2, \quad x \geq 1$

Answer: $\sqrt{x} + 1$

(c) $f(x) = \sqrt{x+4}, \quad x \geq -4$

Answer: $x^2 - 4$

(d) $f(x) = x^3 - 1, \quad \text{all values of } x$

Answer: $\sqrt[3]{x+1}$

7 Find $f^{-1}(x)$.

(a) $f(x) = 3(x-2) + 1$

Answer: $\frac{x+5}{3}$

(b) $f(x) = -2x + 5$

Answer: $\frac{-x + 5}{2}$

(c) $f(x) = (x + 2)^3$

Answer: $\sqrt[3]{x} - 2$

(d) $f(x) = \frac{x + 4}{2}$

Answer: $2x - 4$

8 Find the inverse of each function. In each case there is an x on the top and the bottom of the fraction.

(a) $f(x) = \frac{3x + 1}{x - 2}$

Answer: $\frac{2x + 1}{x - 3}$

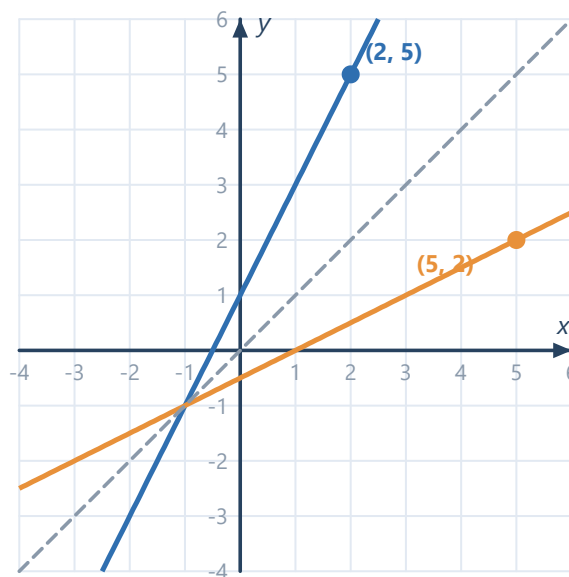
(b) $f(x) = \frac{2x - 1}{x + 3}$

Answer: $\frac{-3x - 1}{x - 2}$

(c) $f(x) = \frac{x}{x + 1}$

Answer: $-\frac{x}{x - 1}$

9 The diagram shows the graph of $y = f(x)$ (blue), the graph of $y = f^{-1}(x)$ (orange) and a dashed line.



(a) Write down the equation of the dashed line.

Answer: $y = x$

(b) Use the diagram to find $f(2)$ and then write down $f^{-1}(5)$.

Answer: $f(2) = 5$ and $f^{-1}(5) = 2$

(c) Find $f^{-1}(x)$ using algebra, where $f(x) = 2x + 1$.

Answer: $\frac{x-1}{2}$

10 The temperature in degrees Fahrenheit is $F = 1.8C + 32$, where C is the temperature in degrees Celsius. Make C the subject of the formula, and use it to convert 68°F to Celsius.

Answer: $C = (F - 32) \div 1.8$; $68^\circ\text{F} = 20^\circ\text{C}$

11 A taxi charges a fixed fee of £3 plus £2.50 per mile. The cost for d miles is $C(d) = 3 + 2.5d$. Find the inverse function, and use it to work out how far Priya can travel for £21.

Answer: $d = (c - 3) \div 2.5$; 7.2 miles

12 Explain why the function $g(x) = x^2$, for all values of x , does not have an inverse function.

Answer: Two inputs (3 and -3) give the same output (9), so the inverse would have two answers

13 $f(x) = 6 \div x$, $x \neq 0$. Show that f is its own inverse, so $f^{-1}(x) = f(x)$.

Answer: $f^{-1}(x) = 6 \div x = f(x)$

14 $f(x) = kx + 6$, where k is a constant. The function f is its own inverse, so $f^{-1}(x) = f(x)$. Find the value of k .

Answer: $k = -1$

15 $f(x) = \frac{2x+3}{x-1}$, $x \neq 1$. Find $f^{-1}(4)$ by solving an equation. Then confirm your answer using the inverse function $f^{-1}(x) = \frac{x+3}{x-2}$.

Answer: $f^{-1}(4) = 3.5$