

GCSE Maths: Iteration and iterative processes (Higher): answers

GCSE Maths - Functions and Advanced Graphs - Higher - Lesson 25.10

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at websink.site/gcse/maths/ch25-functions-advanced-graphs/higher/iteration-and-iterative-processes/

Exercise 25J Iteration and iterative processes Calculator allowed

1 Use each iteration formula and starting value to work out x_1 , x_2 and x_3 . Give your answers to 6 decimal places.

(a) $x_{n+1} = \sqrt[3]{(x_n + 1)}$, $x_0 = 1$

Answer: 1.259921, 1.312294, 1.322354

(b) $x_{n+1} = \frac{3}{x_n^2 + 4}$, $x_0 = 1$

Answer: 0.600000, 0.688073, 0.670624

(c) $x_{n+1} = \frac{x_n^3}{5} + \frac{1}{5}$, $x_0 = 0$

Answer: 0.200000, 0.201600, 0.201639

(d) $x_{n+1} = \sqrt{(3x_n + 2)}$, $x_0 = 3$

Answer: 3.316625, 3.456859, 3.517183

2 Show that each equation has a solution between the two given values.

(a) $x^3 - x - 1 = 0$ between $x = 1$ and $x = 2$

Answer: $f(1) = -1$ and $f(2) = 5$: change of sign

(b) $x^3 + 4x - 3 = 0$ between $x = 0$ and $x = 1$

Answer: $f(0) = -3$ and $f(1) = 2$: change of sign

(c) $x^3 - 5x + 1 = 0$ between $x = 0$ and $x = 1$

Answer: $f(0) = 1$ and $f(1) = -3$: change of sign

(d) $x^2 - 3x - 2 = 0$ between $x = 3$ and $x = 4$

Answer: $f(3) = -2$ and $f(4) = 2$: change of sign

3 Show that each equation can be rearranged to give the iteration formula $x = g(x)$ given.

(a) $x^3 + 2x - 7 = 0$ can be rearranged to $x = \sqrt[3]{(-2x + 7)}$

Answer: Shown

(b) $x^2 - 3x - 2 = 0$ can be rearranged to $x = \sqrt{(3x + 2)}$

Answer: Shown

(c) $x^2 - 3x - 2 = 0$ can be rearranged to $x = \frac{3x + 2}{x}$

Answer: Shown

(d) $x^3 - 5x + 1 = 0$ can be rearranged to $x = \frac{x^3 + 1}{5}$

Answer: Shown

- 4 Use the iteration formula $x_{n+1} = \sqrt[3]{(x_n + 1)}$ with $x_0 = 1.5$ to find a solution of $x^3 - x - 1 = 0$ correct to 3 decimal places. Show that your answer is correct to 3 decimal places.

Answer: 1.325

- 5 Use the iteration formula $x_{n+1} = \frac{3}{x_n^2 + 4}$ with $x_0 = 1$ to find a solution of $x^3 + 4x - 3 = 0$ correct to 3 decimal places. Show that your answer is correct to 3 decimal places.

Answer: 0.674

- 6 Use the iteration formula $x_{n+1} = \sqrt{3x_n + 2}$ with $x_0 = 3$ to find a solution of $x^2 - 3x - 2 = 0$ correct to 3 decimal places. Show that your answer is correct to 3 decimal places.

Answer: 3.562

- 7 Use the iteration formula $x_{n+1} = \frac{x_n^3}{5} + \frac{1}{5}$ with $x_0 = 0$ to find a solution of $x^3 - 5x + 1 = 0$ correct to 3 decimal places. Show that your answer is correct to 3 decimal places.

Answer: 0.202

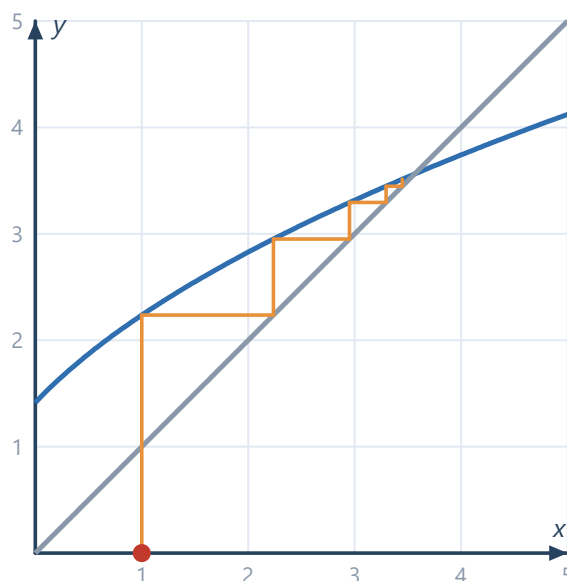
- 8 Use the iteration formula $x_{n+1} = \sqrt[3]{(-2x_n + 7)}$ with $x_0 = 1.5$ to find a solution of $x^3 + 2x - 7 = 0$ correct to 2 decimal places. Show that your answer is correct to 2 decimal places.

Answer: 1.57

- 9 Using $x_{n+1} = x_n^3 - 1$ with $x_0 = 1.5$, work out x_1 , x_2 and x_3 . Does this iteration find the solution of $x^3 - x - 1 = 0$ near 1.32? Explain.

Answer: 2.375, 12.396, 1904.0; no, the terms diverge

- 10 The diagram shows the graph of $y = \sqrt{3x + 2}$ (blue) and $y = x$ (grey). A staircase has been drawn for the iteration $x_{n+1} = \sqrt{3x_n + 2}$ with $x_0 = 1$. Does the iteration converge? Estimate the value it converges to.



Answer: Yes, it converges to about 3.56

- 11** A drug is given in doses of 50 mg. Between doses 30% of the drug in the body is lost. The amount just after each dose is u_n mg, where $u_{n+1} = 0.7u_n + 50$ and $u_0 = 0$. Work out u_1 to u_4 , and use $L = 0.7L + 50$ to find the limiting amount.

Answer: 50, 85, 109.5, 126.65; the limit is 166.7 mg

- 12** A loan of £5000 has interest of 1% added each month and a repayment of £200 is made at the end of each month. The balance after n months is B_n , where $B_{n+1} = 1.01B_n - 200$ and $B_0 = 5000$. Work out B_1 , B_2 and B_3 . After how many months does the balance first fall below £4000?

Answer: £4850.00, £4698.50, £4545.48; after 7 months

- 13** A fish farm has P_n fish at the start of year n . Each year the population grows by 10% and then 50 fish are sold, so $P_{n+1} = 1.1P_n - 50$. Work out P_1 , P_2 and P_3 when

- (a) $P_0 = 600$

Answer: 610, 621, 633.1

- (b) $P_0 = 400$

Answer: 390, 379, 366.9

- 14** For the fish farm in the last question, the amount of fish stays the same if $P_{n+1} = P_n = L$. Find L and explain what happens if the farm starts with more than L fish and with fewer than L fish.

Answer: $L = 500$: more than 500 grows, fewer than 500 falls

- 15** Ravi puts £100 into an account. At the end of each year 3% interest is added and then he adds another £100. Write an iteration formula for the amount u_n after n years with $u_0 = 100$, and find the amount after 3 years.

Answer: $u_{n+1} = 1.03u_n + 100$; £418.36

- 16** The iteration $x_{n+1} = (x_n + 7 \div x_n) \div 2$ with $x_0 = 3$ is used to find $\sqrt{7}$. Work out x_1 , x_2 and x_3 , and show why the limit L is $\sqrt{7}$.

Answer: 2.66667, 2.64583, 2.64575; $L = \sqrt{7} \approx 2.6458$

- 17** $x_{n+1} = 1 + 1 \div x_n$, $x_0 = 1$. Work out x_1 to x_5 . The terms settle on a limit L : show that $L^2 - L - 1 = 0$ and find L using the quadratic formula.

Answer: 2.0000, 1.5000, 1.6667, 1.6000, 1.6250; $L = (1 + \sqrt{5})/2 \approx 1.6180$