

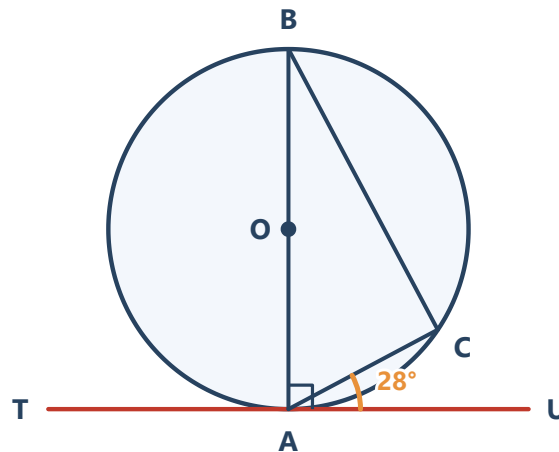
# GCSE Maths: Problems, algebra and proof (Higher): answers

GCSE Maths - Circle Theorems - Higher - Lesson 26.9

**Answers.** Final answers to the worksheet questions. Full step-by-step solutions for every question are free at [websink.site/gcse/maths/ch26-circle-theorems/higher/problems-algebra-and-proof/](https://websink.site/gcse/maths/ch26-circle-theorems/higher/problems-algebra-and-proof/)

## Exercise 26J Multi-step circle theorem problems Non-calculator

- 1  $AB$  is a diameter of a circle with centre  $O$ .  $TU$  is a tangent to the circle at  $A$ .  $C$  is a point on the circle. Angle  $UAC = 28^\circ$ . Give a reason for each answer.



- (a) Work out angle  $CAB$ .

**Answer:**  $62^\circ$  (a tangent is perpendicular to the radius at the point of contact)

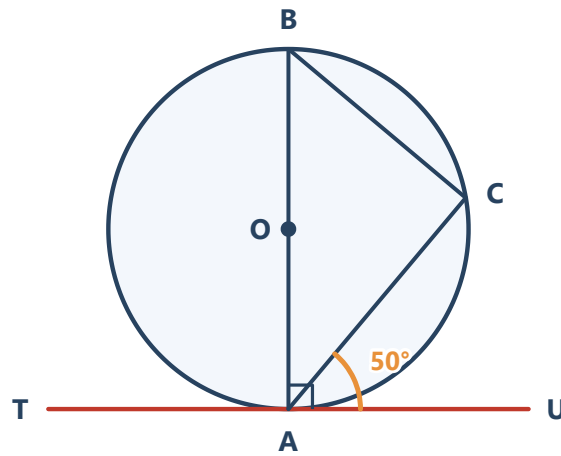
- (b) Work out angle  $ACB$ .

**Answer:**  $90^\circ$  (angle in a semicircle is  $90^\circ$ )

- (c) Work out angle  $ABC$ .

**Answer:**  $28^\circ$  (the angles in a triangle add up to  $180^\circ$ ; the angle between a tangent and a chord is equal to the angle in the alternate segment)

- 2  $AB$  is a diameter of a circle with centre  $O$ .  $TU$  is a tangent to the circle at  $A$ .  $C$  is a point on the circle. Angle  $UAC = 50^\circ$ . Give a reason for each answer.



- (a) Work out angle  $CAB$ .

**Answer:**  $40^\circ$  (a tangent is perpendicular to the radius at the point of contact)

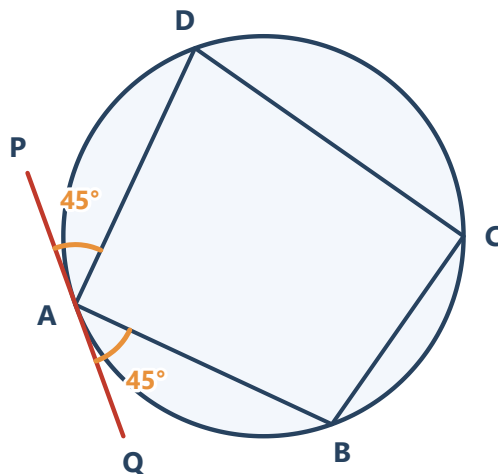
- (b) Work out angle  $ACB$ .

**Answer:**  $90^\circ$  (angle in a semicircle is  $90^\circ$ )

- (c) Work out angle  $ABC$ .

**Answer:**  $50^\circ$  (the angles in a triangle add up to  $180^\circ$ ; the angle between a tangent and a chord is equal to the angle in the alternate segment)

- 3  $A, B, C$  and  $D$  are points on a circle.  $PQ$  is a tangent to the circle at  $A$ . Angle  $QAB = 45^\circ$  and angle  $PAD = 45^\circ$ . Give a reason for each answer.



- (a) Work out angle  $ACB$ .

**Answer:**  $45^\circ$  (the angle between a tangent and a chord is equal to the angle in the alternate segment)

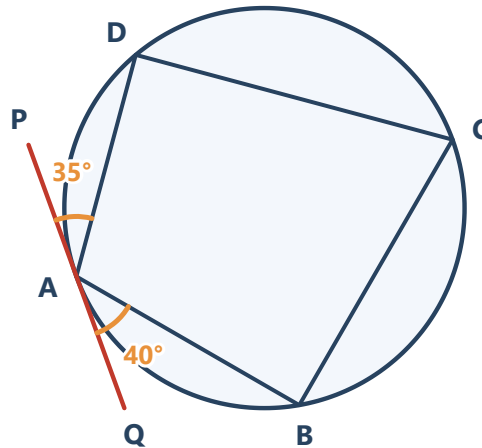
- (b) Work out angle  $BAD$ .

**Answer:**  $90^\circ$  (angles on a straight line add up to  $180^\circ$ )

- (c) Work out angle  $BCD$ .

**Answer:**  $90^\circ$  (opposite angles of a cyclic quadrilateral add up to  $180^\circ$ )

- 4  $A, B, C$  and  $D$  are points on a circle.  $PQ$  is a tangent to the circle at  $A$ . Angle  $QAB = 40^\circ$  and angle  $PAD = 35^\circ$ . Give a reason for each answer.



- (a) Work out angle  $ACB$ .

**Answer:**  $40^\circ$  (the angle between a tangent and a chord is equal to the angle in the alternate segment)

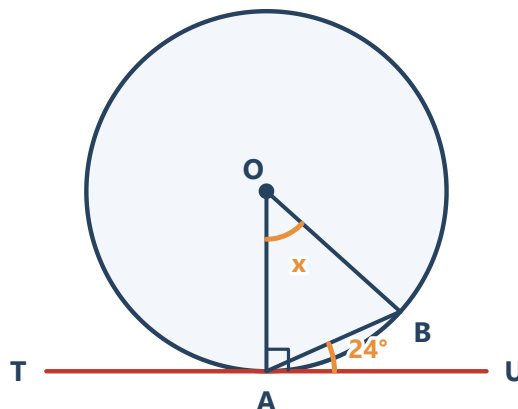
- (b) Work out angle  $BAD$ .

**Answer:**  $105^\circ$  (angles on a straight line add up to  $180^\circ$ )

- (c) Work out angle  $BCD$ .

**Answer:**  $75^\circ$  (opposite angles of a cyclic quadrilateral add up to  $180^\circ$ )

- 5  $TU$  is a tangent to a circle with centre  $O$ , touching the circle at  $A$ .  $B$  is a point on the circle. Angle  $UAB = 24^\circ$ . Give a reason for each step.



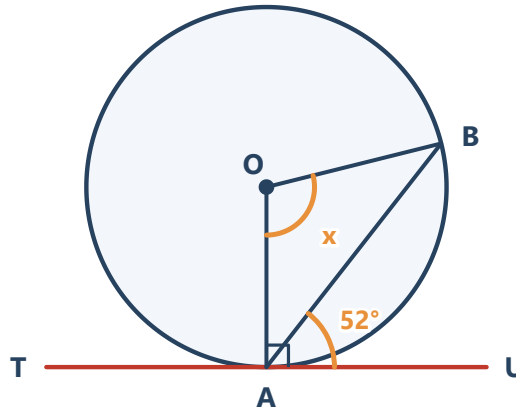
- (a) Work out angle  $OAB$

**Answer:**  $66^\circ$  (a tangent is perpendicular to the radius at the point of contact)

(b) Work out angle  $AOB$

**Answer:**  $48^\circ$  (the base angles of an isosceles triangle are equal; the angles in a triangle add up to  $180^\circ$ )

- 6  $TU$  is a tangent to a circle with centre  $O$ , touching the circle at  $A$ .  $B$  is a point on the circle. Angle  $UAB = 52^\circ$ . Give a reason for each step.



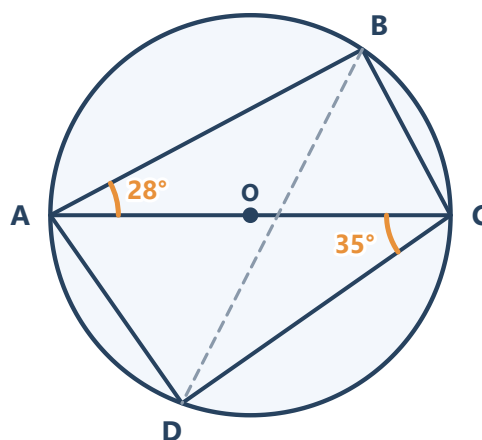
(a) Work out angle  $OAB$

**Answer:**  $38^\circ$  (a tangent is perpendicular to the radius at the point of contact)

(b) Work out angle  $AOB$

**Answer:**  $104^\circ$  (the base angles of an isosceles triangle are equal; the angles in a triangle add up to  $180^\circ$ )

- 7  $A$ ,  $B$ ,  $C$  and  $D$  are points on a circle with centre  $O$ .  $AC$  is a diameter. Angle  $BAC = 28^\circ$  and angle  $ACD = 35^\circ$ . Give a reason for each answer.



(a) Work out angle  $ABC$

**Answer:**  $90^\circ$  (angle in a semicircle is  $90^\circ$ )

(b) Work out angle  $ACB$

**Answer:**  $62^\circ$

(c) Work out angle  $ADB$

**Answer:**  $62^\circ$  (angles in the same segment are equal)

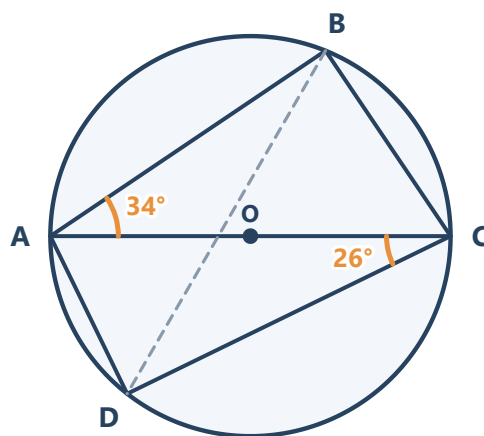
(d) Work out angle  $BCD$

**Answer:**  $97^\circ$

(e) Work out angle  $BAD$

**Answer:**  $83^\circ$  (opposite angles of a cyclic quadrilateral add up to  $180^\circ$ )

- 8  $A, B, C$  and  $D$  are points on a circle with centre  $O$ .  $AC$  is a diameter. Angle  $BAC = 34^\circ$  and angle  $ACD = 26^\circ$ . Give a reason for each answer.



(a) Work out angle  $ABC$

**Answer:**  $90^\circ$  (angle in a semicircle is  $90^\circ$ )

(b) Work out angle  $ACB$

**Answer:**  $56^\circ$

(c) Work out angle  $ADB$

**Answer:**  $56^\circ$  (angles in the same segment are equal)

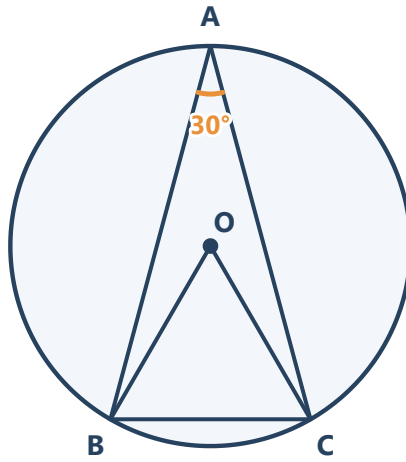
(d) Work out angle  $BCD$

**Answer:**  $82^\circ$

(e) Work out angle  $BAD$

**Answer:**  $98^\circ$  (opposite angles of a cyclic quadrilateral add up to  $180^\circ$ )

- 9  $ABC$  is a triangle with  $AB = AC$ . Its corners are on a circle with centre  $O$ . Angle  $BAC = 30^\circ$ .



- (a) Work out angle  $BOC$ .

**Answer:**  $60^\circ$  (the angle at the centre is twice the angle at the circumference)

- (b) Work out angle  $OBC$ .

**Answer:**  $60^\circ$  (the base angles of an isosceles triangle are equal)

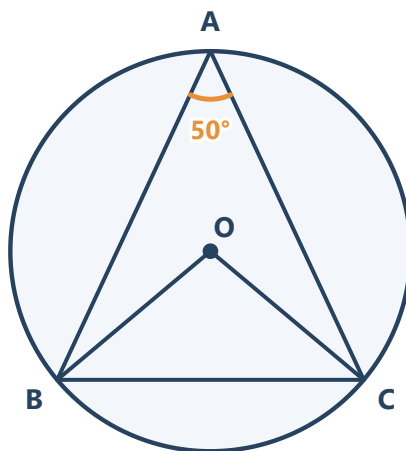
- (c) Work out angle  $ABC$ .

**Answer:**  $75^\circ$  (the base angles of an isosceles triangle are equal)

- (d) Work out angle  $ABO$ .

**Answer:**  $15^\circ$

**10**  $ABC$  is a triangle with  $AB = AC$ . Its corners are on a circle with centre  $O$ . Angle  $BAC = 50^\circ$ .



- (a) Work out angle  $BOC$ .

**Answer:**  $100^\circ$  (the angle at the centre is twice the angle at the circumference)

- (b) Work out angle  $OBC$ .

**Answer:**  $40^\circ$  (the base angles of an isosceles triangle are equal)

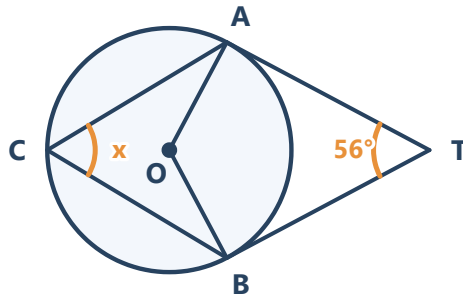
(c) Work out angle  $ABC$ .

**Answer:**  $65^\circ$  (the base angles of an isosceles triangle are equal)

(d) Work out angle  $ABO$ .

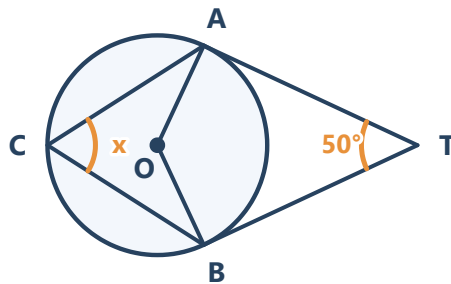
**Answer:**  $25^\circ$

- 11  $TA$  and  $TB$  are tangents to a circle with centre  $O$ .  $C$  is a point on the circle. Angle  $ATB = 56^\circ$ . Work out the size of angle  $ACB$ . Give a reason for each step.



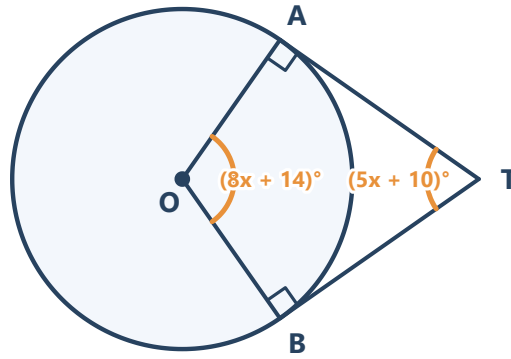
**Answer:**  $62^\circ$  (a tangent is perpendicular to the radius at the point of contact; the angles in a quadrilateral add up to  $360^\circ$ ; the angle at the centre is twice the angle at the circumference)

- 12  $TA$  and  $TB$  are tangents to a circle with centre  $O$ .  $C$  is a point on the circle. Angle  $ATB = 50^\circ$ . Work out the size of angle  $ACB$ . Give a reason for each step.



**Answer:**  $65^\circ$  (a tangent is perpendicular to the radius at the point of contact; the angles in a quadrilateral add up to  $360^\circ$ ; the angle at the centre is twice the angle at the circumference)

- 13  $TA$  and  $TB$  are tangents to a circle with centre  $O$ . Angle  $ATB = (5x + 10)^\circ$  and angle  $AOB = (8x + 14)^\circ$ .



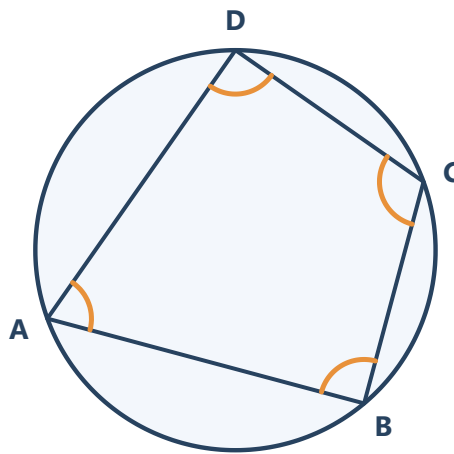
- (a) Work out the value of  $x$ .

**Answer:**  $x = 12$

- (b) Work out the size of angle  $ATB$ .

**Answer:**  $70^\circ$

- 14  $ABCD$  is a cyclic quadrilateral. Angle  $DAB = (4x + 30)^\circ$ , angle  $BCD = (7x + 40)^\circ$ , angle  $ABC = (6y + 18)^\circ$  and angle  $CDA = (3y + 54)^\circ$ . The angles are marked on the diagram.



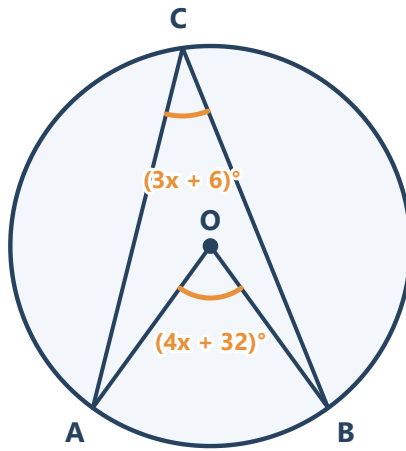
- (a) Work out the value of  $x$ .

**Answer:**  $x = 10$

- (b) Work out the value of  $y$ .

**Answer:**  $y = 12$

- 15  $A$ ,  $B$  and  $C$  are points on a circle with centre  $O$ . Angle  $AOB = (4x + 32)^\circ$  and angle  $ACB = (3x + 6)^\circ$ .



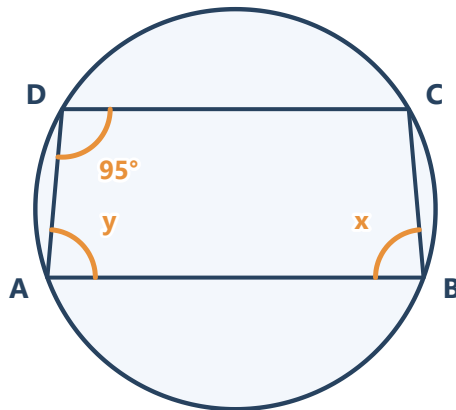
- (a) Work out the value of  $x$ .

**Answer:**  $x = 10$

- (b) Work out the size of angle  $ACB$ .

**Answer:**  $36^\circ$

- 16  $ABCD$  is a cyclic quadrilateral.  $AB$  is parallel to  $DC$ . Angle  $ADC = 95^\circ$ . Give a reason for each step.



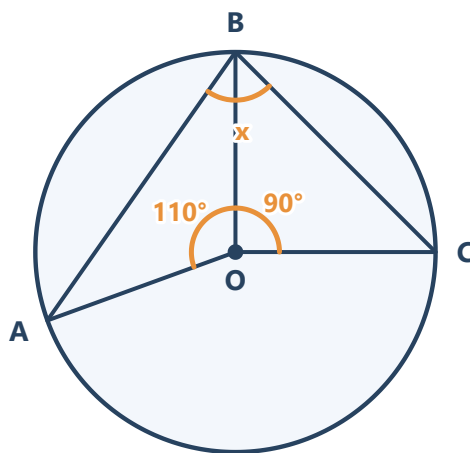
- (a) Work out angle  $ABC$

**Answer:**  $85^\circ$  (opposite angles of a cyclic quadrilateral add up to  $180^\circ$ )

- (b) Work out angle  $BAD$

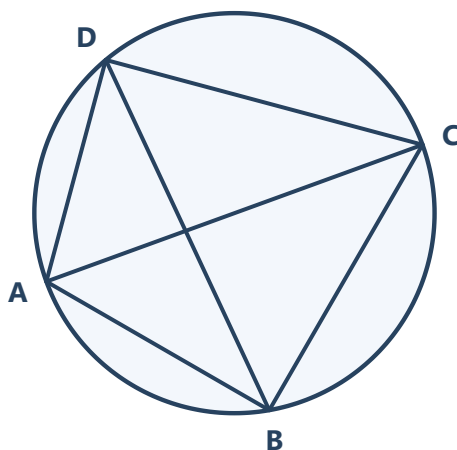
**Answer:**  $85^\circ$  (co-interior angles between parallel lines add up to  $180^\circ$ )

- 17  $A$ ,  $B$  and  $C$  are points on a circle with centre  $O$ . Angle  $AOB = 110^\circ$  and angle  $BOC = 90^\circ$ . Work out the size of angle  $ABC$ . Give a reason for each step.



**Answer:**  $80^\circ$  (the base angles of an isosceles triangle are equal; the angles in a triangle add up to  $180^\circ$ )

- 18**  $ABCD$  is a cyclic quadrilateral. The diagonals  $AC$  and  $BD$  are drawn. Use the circle theorem about angles in the same segment.



- (a) Write down an angle that is equal to angle  $BAC$ .

**Answer:** angle  $BDC$  (angles in the same segment are equal)

- (b) Write down an angle that is equal to angle  $ADB$ .

**Answer:** angle  $ACB$  (angles in the same segment are equal)

- (c) Write down an angle that is equal to angle  $CAD$ .

**Answer:** angle  $CBD$  (angles in the same segment are equal)

- (d) Write down an angle that is equal to angle  $ACD$ .

**Answer:** angle  $ABD$  (angles in the same segment are equal)

## Exercise 26K Reasoning and proof Non-calculator

- 1** Name the circle theorem that is being used in each statement.

- (a)  $AB$  is a diameter and  $C$  is on the circle, so angle  $ACB = 90^\circ$ .

**Answer:** angle in a semicircle is  $90^\circ$

- (b) Angle  $AOB = 120^\circ$  at the centre and angle  $ACB = 60^\circ$  at the circumference.

**Answer:** the angle at the centre is twice the angle at the circumference

- (c) Angle  $BAC =$  angle  $BDC$ , where  $A$  and  $D$  are on the same side of the chord  $BC$ .

**Answer:** angles in the same segment are equal

- (d)  $ABCD$  is a cyclic quadrilateral so angle  $ABC +$  angle  $ADC = 180^\circ$ .

**Answer:** opposite angles of a cyclic quadrilateral add up to  $180^\circ$

- (e)  $TA$  is a tangent and  $OA$  is a radius, so angle  $OAT = 90^\circ$ .

**Answer:** the angle between a tangent and a radius is  $90^\circ$

- (f)  $TA$  and  $TB$  are tangents from  $T$ , so  $TA = TB$ .

**Answer:** tangents from an external point are equal in length

- (g)  $PQ$  is a tangent at  $T$  and angle  $QTA =$  angle  $TBA$ .

**Answer:** the angle between a tangent and a chord is equal to the angle in the alternate segment

- (h)  $M$  is the point where the perpendicular from  $O$  meets the chord  $AB$ , so  $AM = MB$ .

**Answer:** the perpendicular from the centre to a chord bisects the chord

**2** Complete each reason using words from the box: radii, diameter, tangent, cyclic quadrilateral, same segment, semicircle.

- (a) Triangle  $OAB$  is isosceles because  $OA$  and  $OB$  are \_\_\_\_\_.

**Answer:** radii

- (b) Angle  $ABC = 90^\circ$  because  $AC$  is a \_\_\_\_\_.

**Answer:** diameter

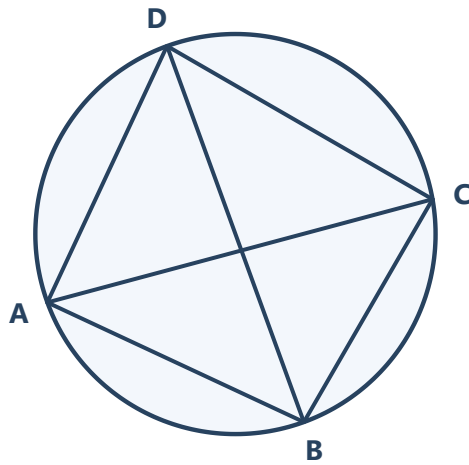
- (c) Angle  $OAT = 90^\circ$  because  $TA$  is a \_\_\_\_\_.

**Answer:** tangent

- (d) Angle  $ABD =$  angle  $ACD$  because they are angles in the \_\_\_\_\_.

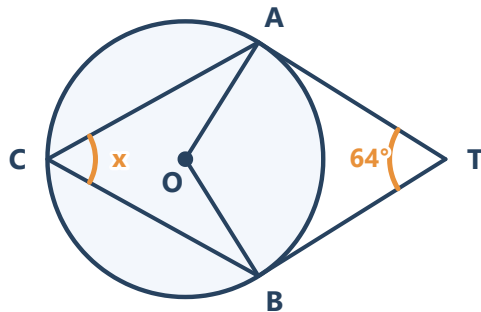
**Answer:** same segment

**3**  $ABCD$  is a cyclic quadrilateral with  $AB = AD$ . The diagonals  $AC$  and  $BD$  are drawn. Prove that  $CA$  bisects angle  $BCD$ .



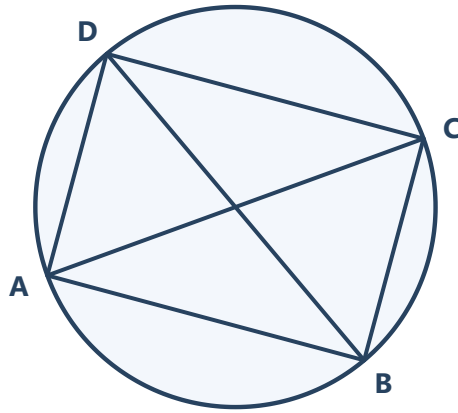
**Answer:** Triangle  $ABD$  is isosceles so angle  $ABD = \text{angle } ADB$ . Angle  $ACD = \text{angle } ABD$  and angle  $ACB = \text{angle } ADB$  (same segment). So angle  $ACD = \text{angle } ACB$ .

- 4  $TA$  and  $TB$  are tangents to a circle and  $C$  is a point on the major arc  $AB$ . Prove that angle  $ACB = 90^\circ - \frac{1}{2}$  angle  $ATB$ .



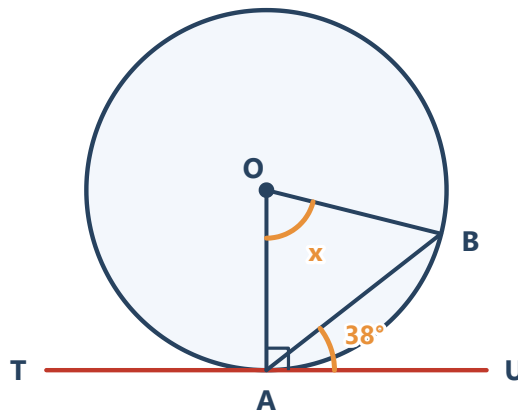
**Answer:** Angle  $AOB = 180^\circ - \text{angle } ATB$  (kite). Angle  $ACB = \frac{1}{2}$  angle  $AOB$  (centre theorem)  $= 90^\circ - \frac{1}{2}$  angle  $ATB$ .

- 5  $ABCD$  is a cyclic quadrilateral with  $AB$  parallel to  $DC$ . Prove that  $AD = BC$ .



**Answer:** Triangles  $ADC$  and  $BCD$  are congruent (AAS), so  $AD = BC$ .

- 6 In the diagram,  $O$  is the centre of the circle, and  $AB$  is a chord. A tangent at  $A$  makes an angle  $x$  with the chord  $AB$ . Show that angle  $AOB = 2x$  in two different ways.



**Answer:** Way 1: alternate segment gives an angle  $x$  at the circumference, and the centre theorem doubles it. Way 2: angle  $OAB = 90^\circ - x$  (tangent and radius), then angle  $AOB = 180^\circ - 2(90^\circ - x) = 2x$ .