

GCSE Maths: The alternate segment theorem (Higher): answers

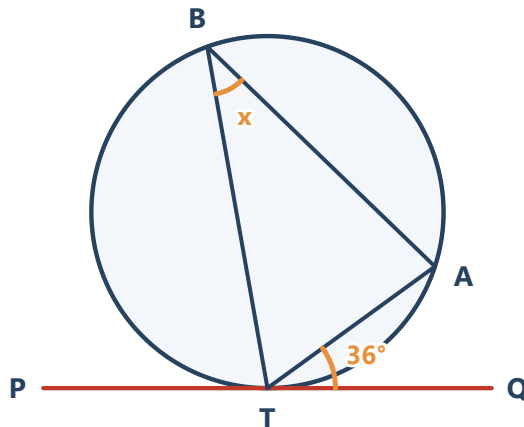
GCSE Maths - Circle Theorems - Higher - Lesson 26.7

Answers. Final answers to the worksheet questions. Full step-by-step solutions for every question are free at websink.site/gcse/maths/ch26-circle-theorems/higher/the-alternate-segment-theorem/

Exercise 26G The alternate segment theorem

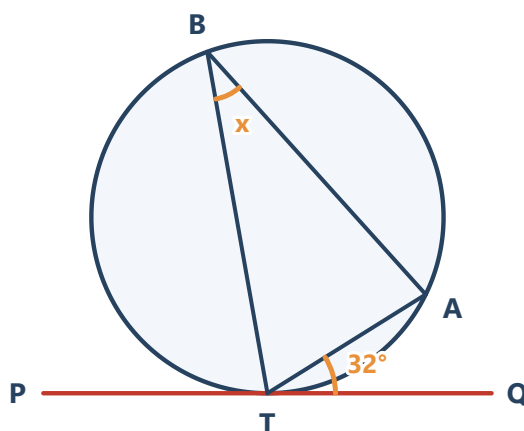
Non-calculator

- 1 PQ is a tangent to the circle at T . A and B are points on the circle. Angle $QTA = 36^\circ$. Work out the size of angle TBA . Give a reason for your answer.



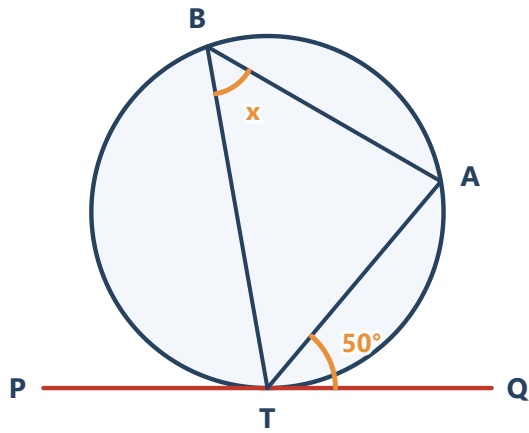
Answer: 36° (the angle between a tangent and a chord is equal to the angle in the alternate segment)

- 2 PQ is a tangent to the circle at T . A and B are points on the circle. Angle $QTA = 32^\circ$. Work out the size of angle TBA . Give a reason for your answer.



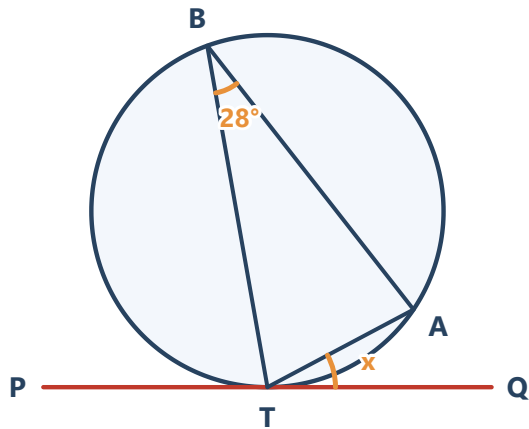
Answer: 32° (the angle between a tangent and a chord is equal to the angle in the alternate segment)

- 3 PQ is a tangent to the circle at T . A and B are points on the circle. Angle $QTA = 50^\circ$. Work out the size of angle TBA . Give a reason for your answer.



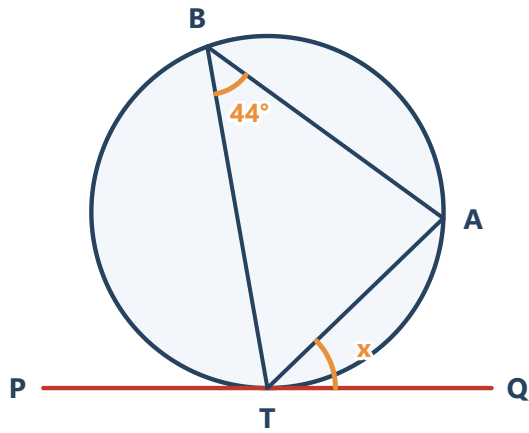
Answer: 50° (the angle between a tangent and a chord is equal to the angle in the alternate segment)

- 4 PQ is a tangent to the circle at T . A and B are points on the circle. Angle $TBA = 28^\circ$. Work out the size of angle QTA . Give a reason for your answer.



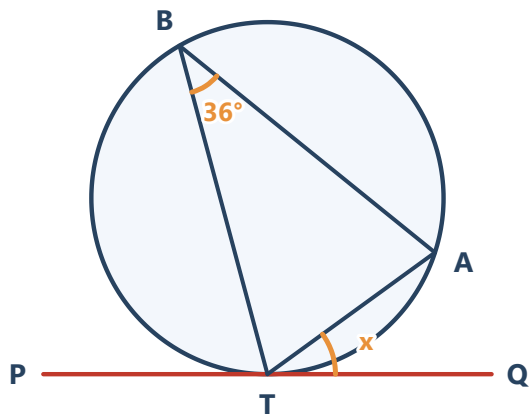
Answer: 28° (the angle between a tangent and a chord is equal to the angle in the alternate segment)

- 5 PQ is a tangent to the circle at T . A and B are points on the circle. Angle $TBA = 44^\circ$. Work out the size of angle QTA . Give a reason for your answer.



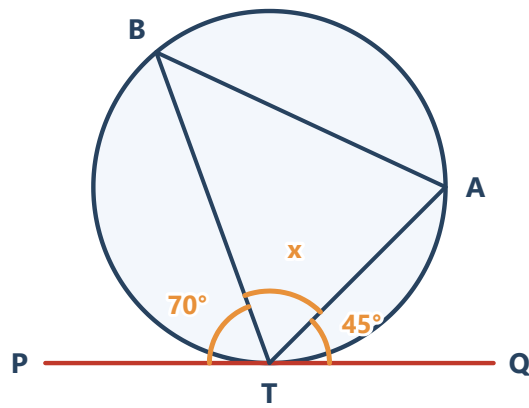
Answer: 44° (the angle between a tangent and a chord is equal to the angle in the alternate segment)

- 6 PQ is a tangent to the circle at T . A and B are points on the circle. Angle $TBA = 36^\circ$. Work out the size of angle QTA . Give a reason for your answer.



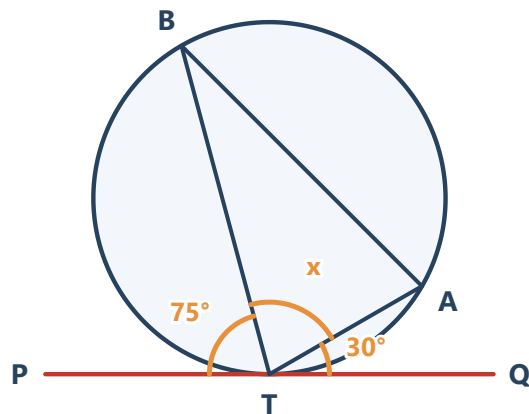
Answer: 36° (the angle between a tangent and a chord is equal to the angle in the alternate segment)

- 7 PQ is a tangent to the circle at T . A and B are points on the circle. Angle $QTA = 45^\circ$ and angle $PTB = 70^\circ$. Work out the size of angle ATB . Give a reason for your answer.



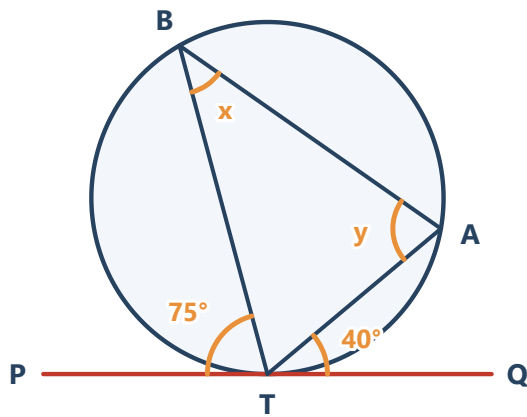
Answer: 65° (angles on a straight line add up to 180°)

- 8 PQ is a tangent to the circle at T . A and B are points on the circle. Angle $QTA = 30^\circ$ and angle $PTB = 75^\circ$. Work out the size of angle ATB . Give a reason for your answer.



Answer: 75° (angles on a straight line add up to 180°)

- 9 PQ is a tangent to the circle at T . A and B are points on the circle. Angle $QTA = 40^\circ$ and angle $PTB = 75^\circ$. Give a reason for each answer.



- (a) Work out angle TBA

Answer: 40° (the angle between a tangent and a chord is equal to the angle in the alternate segment)

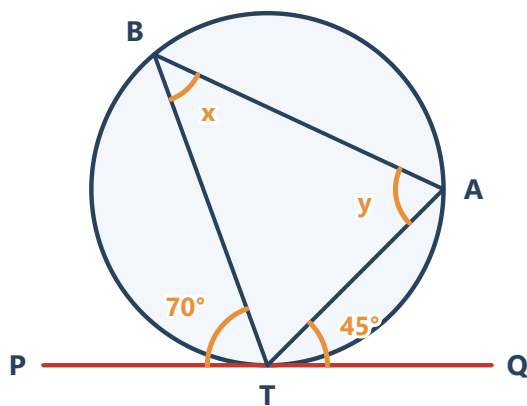
- (b) Work out angle TAB

Answer: 75° (the angle between a tangent and a chord is equal to the angle in the alternate segment)

- (c) Work out angle ATB

Answer: 65° (the angles in a triangle add up to 180°)

- 10** PQ is a tangent to the circle at T . A and B are points on the circle. Angle $QTA = 45^\circ$ and angle $PTB = 70^\circ$. Give a reason for each answer.



- (a) Work out angle TBA

Answer: 45° (the angle between a tangent and a chord is equal to the angle in the alternate segment)

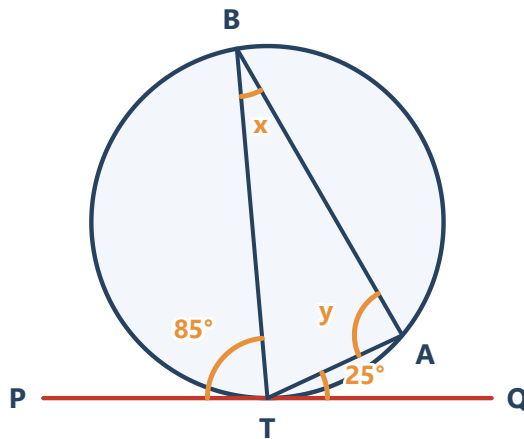
- (b) Work out angle TAB

Answer: 70° (the angle between a tangent and a chord is equal to the angle in the alternate segment)

- (c) Work out angle ATB

Answer: 65° (the angles in a triangle add up to 180°)

- 11 PQ is a tangent to the circle at T . A and B are points on the circle. Angle $QTA = 25^\circ$ and angle $PTB = 85^\circ$. Give a reason for each answer.



- (a) Work out angle TBA

Answer: 25° (the angle between a tangent and a chord is equal to the angle in the alternate segment)

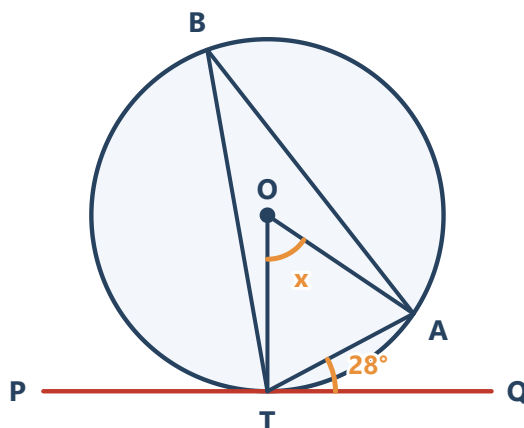
- (b) Work out angle TAB

Answer: 85° (the angle between a tangent and a chord is equal to the angle in the alternate segment)

- (c) Work out angle ATB

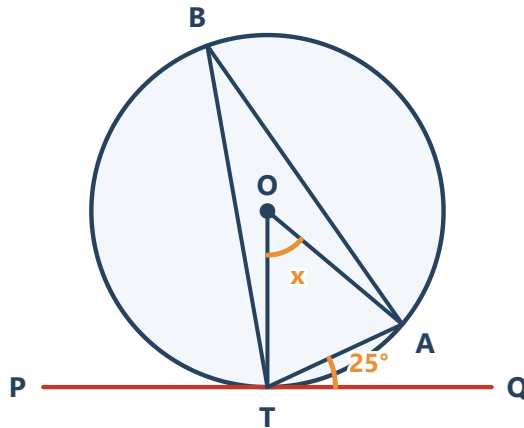
Answer: 70° (the angles in a triangle add up to 180°)

- 12 PQ is a tangent at T to a circle with centre O . Angle $QTA = 28^\circ$. Work out the size of angle TOA . Give a reason for each step.



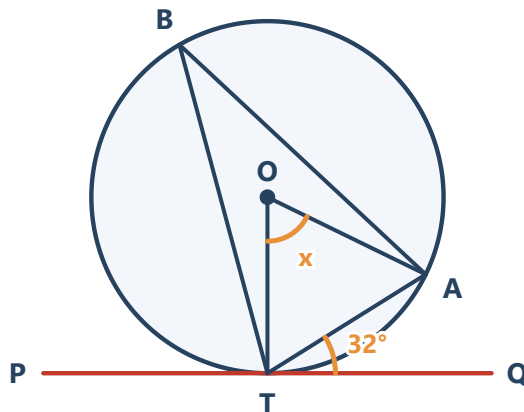
Answer: 56° (the angle between a tangent and a chord is equal to the angle in the alternate segment; the angle at the centre is twice the angle at the circumference)

- 13** PQ is a tangent at T to a circle with centre O . Angle $QTA = 25^\circ$. Work out the size of angle TOA . Give a reason for each step.



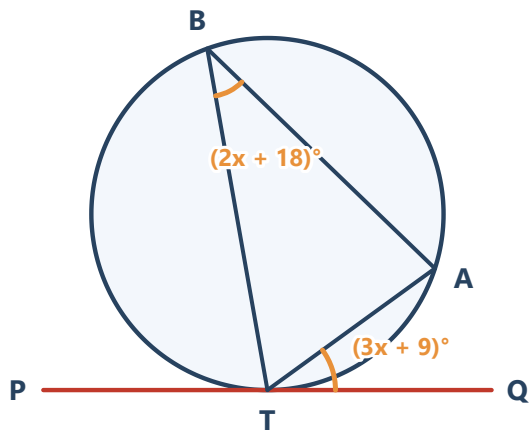
Answer: 50° (the angle between a tangent and a chord is equal to the angle in the alternate segment; the angle at the centre is twice the angle at the circumference)

- 14** PQ is a tangent at T to a circle with centre O . Angle $QTA = 32^\circ$. Work out the size of angle TOA . Give a reason for each step.



Answer: 64° (the angle between a tangent and a chord is equal to the angle in the alternate segment; the angle at the centre is twice the angle at the circumference)

- 15** PQ is a tangent to the circle at T . Angle $QTA = (3x + 9)^\circ$ and angle $TBA = (2x + 18)^\circ$. The two angles are marked on the diagram.



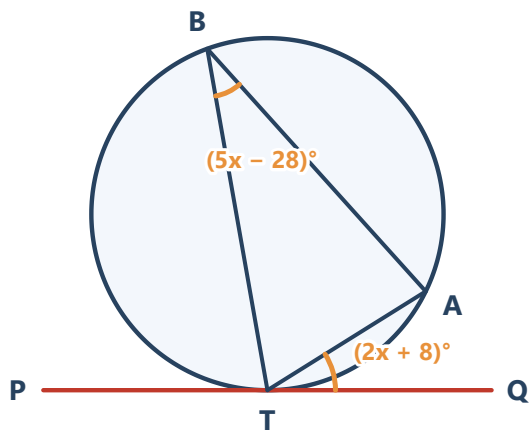
- (a) Work out the value of x .

Answer: $x = 9$

- (b) Work out the size of angle QTA .

Answer: 36°

- 16** PQ is a tangent to the circle at T . Angle $QTA = (2x + 8)^\circ$ and angle $TBA = (5x - 28)^\circ$. The two angles are marked on the diagram.



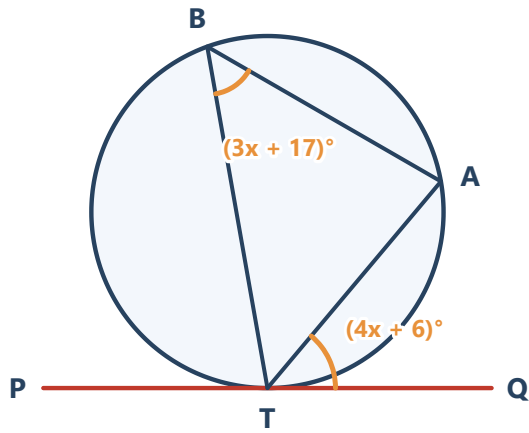
- (a) Work out the value of x .

Answer: $x = 12$

- (b) Work out the size of angle QTA .

Answer: 32°

- 17** PQ is a tangent to the circle at T . Angle $QTA = (4x + 6)^\circ$ and angle $TBA = (3x + 17)^\circ$. The two angles are marked on the diagram.



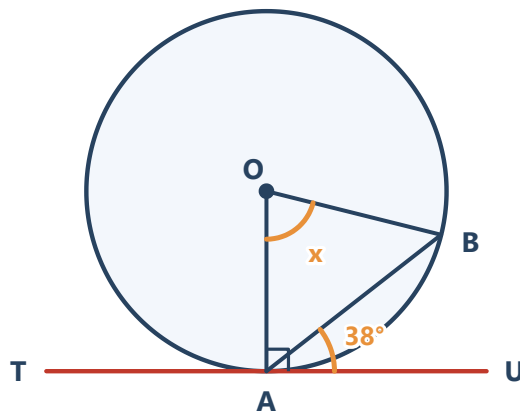
- (a) Work out the value of x .

Answer: $x = 11$

- (b) Work out the size of angle QTA .

Answer: 50°

- 18** TU is a tangent to a circle with centre O , touching the circle at A . B is a point on the circle. Angle $UAB = 38^\circ$. Give a reason for each step.



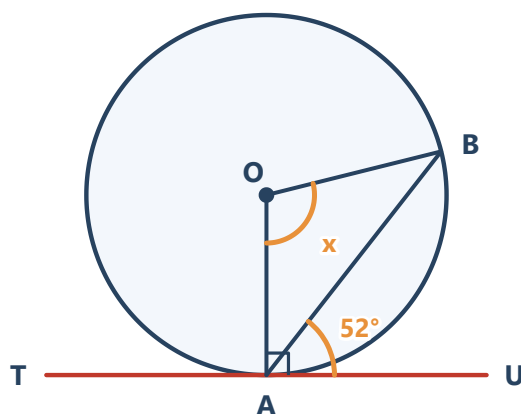
- (a) Work out angle OAB

Answer: 52° (a tangent is perpendicular to the radius at the point of contact)

- (b) Work out angle AOB

Answer: 76° (the base angles of an isosceles triangle are equal; the angles in a triangle add up to 180°)

- 19** TU is a tangent to a circle with centre O , touching the circle at A . B is a point on the circle. Angle $UAB = 52^\circ$. Give a reason for each step.



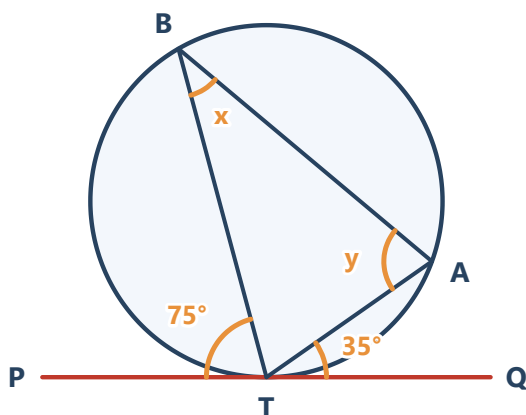
- (a) Work out angle OAB

Answer: 38° (a tangent is perpendicular to the radius at the point of contact)

- (b) Work out angle AOB

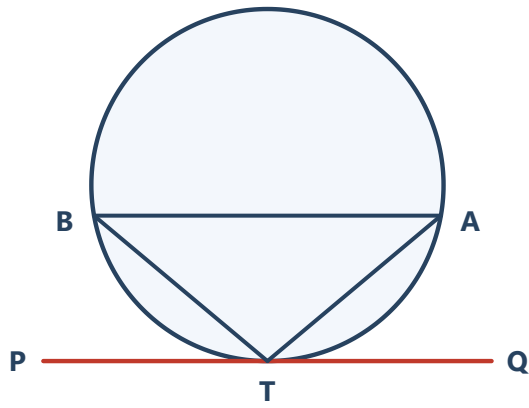
Answer: 104° (the base angles of an isosceles triangle are equal; the angles in a triangle add up to 180°)

- 20 Kai says: "By the alternate segment theorem, angle $PTB = \text{angle } TBA$." Is Kai right? Explain.



Answer: No. Angle PTB is between the tangent and the chord TB , so it equals the angle in the alternate segment of that chord, which is angle TAB .

- 21 PQ is a tangent to a circle at T . A and B are points on the circle so that the chord AB is parallel to PQ . Prove that triangle TAB is isosceles.



Answer: Angle QTA = angle TAB (alternate angles) and angle QTA = angle TBA (alternate segment), so angle TAB = angle TBA and $TA = TB$.